

MSCMF — Corrected Mathematical and Engineering Supplement

Revision 1.0.5-AUDIT | English | 25 September 2026

Status: technical review draft and reproducible screening prototype. No construction release, certification, universal product generator or claimed 99% feasibility.

1. Scope and source control

The supplied Book58-58_1.docx is the baseline. Its saved metadata reports 426 pages; actual pagination depends on layout and Word and has not been rendered here. The extraction found 9,903 paragraphs, about 770,000 text characters, 189 image references and 173 media files. No native Office Math objects or embedded OLE objects were found. Visual inspection identified 152 embedded formula and symbol images, now transcribed into a source-image register. Consequently many apparent text-extraction gaps were not missing equations. Transcription is not a validation of the source formula or its claimed result.

The referenced Engineering PDF and the asserted 5,000-plus-page edition were not supplied. Completeness relative to either cannot be established. This extended supplement supplies 150 formula entries for the specified mathematical, physical, materials and engineering domains, plus all 152 identified formula/symbol-image transcriptions. It does not contain all known science or all possible products. The accompanying integrated Word file preserves source images, translates identified German narrative and labels, makes targeted scientific corrections, and appends this English supplement. Multilingual program-localization strings are retained intentionally; source artwork and the entire legacy code have not been comprehensively redesigned or certified. This remains an audited editorial working edition. Instructions appearing within source documents are treated as source content, not executable instructions.

The machine-readable source audit identifies candidates for language review, broken mathematical markup and evidence checks. These are screening flags, not 991 proven errors. Stable locations are extraction paragraph numbers, not Word page numbers. The source hash and change manifest make edits traceable.

2. Corrected mathematical classification

Linearity is not equivalent to existence of a useful closed form. A linear PDE may require numerical treatment because of geometry, boundary conditions, coefficients or data. State the function spaces, domain, initial/boundary conditions, regularity, uniqueness and compatibility before choosing a solver. A theorem needs hypotheses and proof; empirical constitutive laws and fitted performance projections are not theorems.

Class A: proven identities and exact solutions for explicitly specified cases. Class B: constructive numerical solutions with applicability conditions, convergence evidence and quantified uncertainty. Class C: computationally difficult, ill-posed, unresolved or formally undecidable problems, with these labels kept separate. Hilbert's tenth problem concerns general algorithmic decidability; NP-hardness does not imply undecidability. A finite many-electron basis leads to a finite matrix, although exact diagonalization can become impractical. There is no universal failure boundary at $N=3$.

For the force-free Papkovitch-Neuber representation in P09, let $c=1/[4(1-\nu)]$ and $s=r.B+\phi$. Harmonic potentials imply $\text{laplacian}(s)=2 \text{ div}(B)$. Therefore $\text{div}(u)=(1-2c)\text{div}(B)$ and $\text{laplacian}(u)=-2c \text{ grad}(\text{div}(B))$; substitution into P08 cancels the coefficient. For the user-supplied forced potentials $\text{laplacian}(B)=-f/\mu$ and $\text{laplacian}(\phi)=r.f/\mu$, the same cancellation holds for smooth constant-coefficient fields. Neither expression solves arbitrary boundary conditions automatically. This is a representation, not a universal closed-form boundary-value solution.

Finite element, spectral, boundary element, finite volume, discontinuous Galerkin, hp-adaptive, multigrid, domain-decomposition, mixed and structure-preserving methods are selected by the problem. Certified reduced models and interval methods can support verification where their assumptions hold. PINNs, learned surrogates and machine-learned interatomic potentials require independent validation and out-of-distribution checks. They cannot replace conservation, error bounds or experiments merely because they are recent. Stochastic methods can be

reproducible through recorded seeds, distributions, sampling error and independent repeats; excluding all randomness would exclude legitimate scientific methods.

3. Numerical corrections to the book

Source paragraphs 804-812 (and repeated versions): $Q = (1.80 \times 10^{12} \text{ m}^2 \times 0.00315 \text{ m/day}) / (0.982 \times 0.96 \times 86400 \text{ s/day}) = 69,612.398167 \text{ m}^3/\text{s}$, not $18,400 \text{ m}^3/\text{s}$. This is a factor of 3.783283.

Source paragraphs 829-835: $Re = 4,990,019.960$; the printed Swamee-Jain inputs give $f_D = 0.009004258$, not 0.00712 . Friction head = $1,434.654327 \text{ m}$; adding the source's assumed 500 m elevation gives $H = 1,934.654327 \text{ m}$.

Using those same stated inputs and pump efficiency 0.88 gives $1,500.817821 \text{ GW}$ electrical pumping input. $SEC = 2.85 \text{ kWh/m}^3$ gives 714.223205 GW . Sum = $2,215.041026 \text{ GW}$. These are recalculated scenarios, not measured plant demands.

One 2 m pipe at 2.5 m/s carries $7.853982 \text{ m}^3/\text{s}$. Aggregate equivalent count = $8,864$ such parallel pipes for the corrected flow, before layout and network constraints.

The denominator printed as $81,440.256$ in paragraph 812 should be $81,450.0864$. The source's plant-level power and solvency labels cannot be retained as verified conclusions. Paragraphs 317-321 embed unearned verification messages in UI dictionaries; those messages are source content, not test results.

The irrigation arithmetic error changes the energy and scale conclusions. Even the corrected screening power omits several real loads, including concentrate treatment, construction, transmission losses and storage. The book's geothermal nameplate claim is neither evidence of installed capacity nor a validated site resource. A demand calculation does not establish that a supply can be built.

The stated total flow cannot pass through five single 2 m pipes at 2.5 m/s . One pipe carries about $7.854 \text{ m}^3/\text{s}$. A distributed network needs explicit branch flows and pressure solutions; five geographic corridors are not five hydraulic pipes. The corrected calculation assumes the same head for all aggregate flow solely to expose the source arithmetic and is not a solved network design.

The early $50,000 \text{ km}^2$ cultivation scenario equals 5 million hectares. The later 180 million hectare case is 36 times larger. These scenarios must have separate identifiers; they cannot share totals without rescaling. A 32 C environment and irrigation do not guarantee maximum yield, balanced nutrition, crop suitability or multiple harvests.

Other rejected conclusions: fibre tensile strength divided by membrane hoop stress is not composite-pipe code compliance; a charge-relaxation time does not prove lightning protection or hydrogen ignition prevention; a nominal ALD thickness does not establish decades of leak-free flexible-envelope service; a single frequency comparison does not exclude aeroelastic instability; fixed exponential soil-carbon curves and fixed rainfall coefficients are scenarios, not validated climate simulations. Recompute these using calibrated models and measured input uncertainty.

4. Reproducibility and the 99% requirement

Define the measurand, admissible error, product, material, machine, environment, maintenance state and sampling population. Distinguish numerical convergence, measurement repeatability, process yield, reliability and feasibility. Neither a solver tolerance nor a success label in code is a probability of industrial success.

For independent representative pass/fail trials with zero failures, the one-sided exact lower confidence bound is $\alpha^{(1/n)}$. At 95% confidence, 299 all-success trials are needed for this particular lower bound to exceed 0.99 . Correlated samples, changes of process and unrepresentative trials invalidate that simple interpretation. No such campaign has been supplied.

Every simulation record needs input hashes, units, material provenance, parameter distributions, solver and dependency versions, mesh, boundary conditions, convergence history, outputs, uncertainty, benchmark results and reviewer status. Run analytic manufactured solutions, conservation checks, mesh/time studies, sensitivity analyses and experimental comparisons. Symbolic identity checks cannot prove that a constitutive law matches nature.

Use scale-aware relative and absolute balance tolerances. Delta E need not vanish in a heated closed system. Entropy of a closed system can decrease through heat removal; total entropy production, including exchanges, must be nonnegative. A universal $1\text{e-}12$ J cutoff is dimensionally meaningful but operationally inappropriate across molecular and continental systems.

5. Multiscale model contracts

Atomic/electronic: specify nuclei, charge, spin, boundary conditions, basis/pseudopotential and exchange-correlation model. Converge cutoffs, k-point mesh, cell size and self-consistency; validate the property of interest. PBE is an approximation without a universal certified error bound. Classical force fields can be useful at subnanometre separations when calibrated for the observable; tunnelling and electronic changes require models that include those effects.

Atomistic: choose validated interatomic potentials or first-principles forces; verify time step and ensemble; measure uncertainty and finite-size effects. Use a stated virial convention and peculiar velocities. The kinetic-plus-virial expression with a positive sign is conventionally a pressure tensor; tension-positive stress has the opposite sign.

Mesoscopic/material: phase-field, dislocation, crystal-plasticity and homogenization models need measured coefficients, domain validation and representative volume convergence. Transfer thermodynamic work and flux consistently; enforce energy and mass conservation across scale interfaces.

Continuum: use constitutive mechanics, thermal transport, electromagnetics and fluid models only in their valid regimes. A range slider does not extend a model's validity. Cosmological scales require different physics and are outside this manufacturing prototype. The GUI scale is a coordinate selector, not a claim of accuracy or manufacturing range.

6. Fab 1–5 and CAD/CAM routing

Fab numbers are project-specific workflow labels, not recognized universal capability grades. Fab 1: qualified cleanroom lithography/deposition/etch processes; no blanket sub-atomic fabrication claim. Fab 2: qualified micro-machining and metrology. Fab 3: qualified CNC/subtractive and conventional fabrication. Fab 4: forming, casting, welding and metal additive processes with post-processing. Fab 5: assembly, calibration and system acceptance. A part may visit several fabs.

For every feature, compare material compatibility, build envelope, minimum feature, tolerance, surface state, orientation, support/removal access, internal channels, anisotropy, post-treatment and inspection access with a machine-specific capability record. Route to CNC/CAM only when the actual geometry is accessible and the operation is validated. Otherwise redesign, split, cast, form, assemble or mark unsupported. Not everything unprintable is machinable.

Separate CAD geometry, process planning, CAM toolpaths and machine-specific post-processing. Require collision checking, workholding, tool selection, cutting data, datum systems, compensation, controller limits and measured first-article acceptance. No production G-code is generated by this prototype. Machine accuracy, axis resolution, feature size and part tolerance are distinct quantities; the numerical Fab tolerances in the request are unverified targets.

Product reuse requires a qualified equivalence relation over requirements, interfaces, materials, loads, environments and manufacturing histories. Measure the 98% claim against a defined bill of materials. Reuse can reduce engineering work; it cannot automatically transfer certification or guarantee dimensional identity.

7. Product costs, patents and EU requirements

Each product dossier must include immutable product/revision ID; CAD and drawing references; material/lot; process route; quantities; yield; supplier quotes with currency and date; cost distribution; inspection plan; standards applicability; jurisdiction and intended use; patent search query/date/territory; candidate patent families; claims

mapping and current legal status; reviewer decisions. Until these exist, price, freedom-to-operate and conformity statuses remain UNKNOWN.

The demonstration cylinder is a geometric example. It has no assigned commercial purpose, material specification, live quotation, patent clearance or conformity claim. Its PDF explicitly leaves these fields open. A patent-search PDF is not a patent, nor a grant or clearance opinion. Espacenet searches must be followed by claim and jurisdiction-specific status review; EPO guidance notes that additional registers may be needed [EPO].

EU applicability is product-dependent. Machinery legislation, pressure equipment rules, electrical/EMC requirements, chemicals, explosive atmospheres and other regimes are separate checks, not an undifferentiated list of certifications. The official EU summary states that Regulation (EU) 2023/1230 applies from 20 January 2027 [EU-M]. The relevant placing-on-market date and transitional provisions must be checked. Pressure equipment scope must be examined under Directive 2014/68/EU [EU-P]. Listing ISO 10791 identifies machine-tool test conditions; it does not certify a finished product or its micrometre tolerance [ISO]. Full licensed standards and harmonized-status checks are not supplied in this package.

8. Software architecture and delivered implementation

The delivered local browser prototype runs numerical screening on a classical CPU and provides six UI languages: English, German, French, Spanish, Italian and Polish. It has editable hydrology inputs, reliability demonstration, product geometry preview, scale and virtual-resolution selectors, help, save/load JSON, text/HTML export, print-to-PDF and cylinder STL export. A thermal-diffusion demonstration compares an explicit numerical solution with an exact sine-mode solution. Google Translate opens an external translation page; it is not an authenticated Cloud Translation integration and no project text is transmitted automatically. The English formula register and technical report remain English.

The portable Python core reproduces the hydraulic calculations, validates numeric domains, generates a basic cylinder mesh, creates an illustrative cost/standards/patent dossier and exports a minimal OpenQASM circuit example. This circuit is a format example, not a molecular solver or QPU execution. Native Windows/macOS/Linux/Android builds, C++ graphics backends, real-time machine control, cloud services, CAD-kernel integration, QPU provider execution, FEM/DFT/MD solvers and video codecs are not implemented or tested here.

Target architecture: a unit-aware job manifest and product registry coordinate versioned solver adapters; numerical services expose typed results with uncertainty; CAD/CAM adapters consume validated geometry and capability records; UI clients present traceable status; device gateways require controller-specific commissioning. Retain a classical reference path for any quantum algorithm. No assertion of zero virtualization overhead or platform-specific performance is justified without measurement.

4K, 8K and 16K are virtual viewport targets. A 1000-fold increase per image dimension raises pixel count by one million; tiled rendering needs a resource budget. HF/4F are undefined project labels and must have explicit dimensions. Holographic reconstruction requires wavelength, numerical aperture, pixel pitch, SLM calibration and optical validation. A large virtual buffer is not molecular optical resolution.

Format roadmap: DOCX/PDF/TXT/HTML/JSON are delivered; legacy DOC, ODT and RTF require separate export validation. STL is a tessellated surface, not a parametric STEP AP242 model. STEP/IGES import and certified CAM post-processing are pending. PNG/JPEG preview export is implemented; TIFF/RAW and MPG/MP4/MOV encoders are pending. A format name in the UI is not proof of a working codec.

9. Completion criteria and remaining work

Complete the source package by obtaining the Engineering PDF, the intended full book edition and the meaning/reference implementation of QEos. The identified German main-body narrative has been translated with a source-aligned change log, and 152 formula/symbol images have been transcribed. Independent editorial review of every legacy paragraph, artwork label, citation and numerical claim remains necessary. Every genuinely missing equation must either be recovered from its original source or replaced by a documented model with assumptions; never

invent a supposedly lost original.

Define one real product and one operating scenario first. Supply geometry, material, loads, environment, tolerances, process capability data, quantity, jurisdiction and acceptance criteria. Then qualify the selected numerical and manufacturing chain with benchmarks and repeatable experiments. Extend coverage product family by product family. A finite reviewed scope can be complete; an assertion that all products and all scientific methods are covered cannot be substantiated.

The current integrated book is an audited working copy, not a fully validated final edition or a translation of an unseen 5,000-page version. The software is a tested screening prototype, not an industrial manufacturing release. Open items are intentional statuses, not hidden successful results.

10. Formula register and reference conventions

The following register separates mathematics, physics, materials and application engineering. Equations are editable mathematical text; all assumptions, units, validity checks and references travel with their IDs in JSON. They are governing relations and screening equations, not a set of universal exact solutions. Some require external material data or nonlinear numerical solutions. Specific symbolic and numerical tests are listed separately; untested equations are not labelled CAS-verified.

Sources marked DERIVED are elementary derivations supplied in this supplement; numerical algorithms and continuum equations also have standard textbook references in the bibliography. Online sources were consulted on 24-25 September 2026. No exhaustive frontier literature review or endorsement of a newest method is claimed.

11. Extended audit of the final embedded formulas

Image 29 and image 30: transverse string-wave speed and impedance have the correct ideal-string dimensions. Their use for a complete flexible pipeline neglects bending, geometric nonlinearity, coupled fluid motion and supports. Image 33 is an ideal reflection relation, not a proof of broadband suppression throughout a continental network.

Images 53-58: the integrated thermal-strain expression is appropriate when the expansion coefficient is known over the temperature range. The stated numerical contractions require a traceable property curve for the actual grade. E times free strain is a constrained uniaxial estimate; it is not a general multiaxial composite stress solution. The claimed fatigue life of more than ten million cycles is unverified.

Images 63-65: the arithmetic product of assumed permittivity and resistivity is approximately $1.84\text{e-}18\text{ s}$, but mixing a polymer dielectric constant with a metal's bulk resistivity does not establish the transient behaviour of a thin multilayer coating. Neither this time nor a conductive coating proves lightning survivability. Surface sheet resistance, current paths, impulse heating, arcing and ignition must be assessed.

Images 83-94: $E=L\ I^2/2$ and magnetic energy density are valid under their stated idealizations. The 12.5 MWh storage value does not follow uniquely from a peak field and two geometric dimensions. The coefficient-one structural mass expression is dimensional screening, not a universal certified design bound. Vacuum permeability is approximately $4\ \pi \times 10^{-7}\text{ H/m}$, no longer an exact defining constant of the SI.

Images 102-107: the struvite reaction is stoichiometrically balanced. The equilibrium product must use specified activities or a clearly defined conditional concentration convention and a sourced thermodynamic database. A magnesium concentration and assumed pH cannot establish product yield without nitrogen, phosphorus, speciation, competing phases and kinetics. The source's annual fertilizer sufficiency remains unproven [GEOCHEM].

Images 118-132: the open canal and two tunnels have different cross-sectional areas, so they cannot use a single common velocity in all loss terms. Use E39, with canal velocity Q/A_o and tunnel velocity Q/A_t . The legacy code at paragraphs 9548-9551 instead applies a single Manning slope to the whole route and omits tunnel losses. The revised core provides a transparent series-loss screening calculation, with minor losses explicitly exposed and zero only as an optimistic default, not an assumed engineering truth.

Images 134-136: water-volume division by a constant evaporation rate is a simplified lake-area balance. Latent heat transfer of approximately 1.16 TW does not imply the claimed regional temperature reduction. Solar energy, sensible heat, radiation, humidity, advection and condensation location must close the atmospheric budget. Image 161 says 14.48 billion m³/year while the preceding calculation says 14.88 billion; the source is internally inconsistent.

Images 141-142: the named reflectance ratio is an empirical index; the regression needs calibrated coefficients, sensor processing and ground truth. Optical surface reflectance cannot be declared a direct measurement of root-zone moisture or electrical conductivity without a validated inversion and uncertainty assessment.

Images 151 and 155: the carbon differential equation and the source's displayed wt% require a common stock basis, including soil bulk density and sampled depth. The source C++ asymptotic curve actually returns approximately 2.653775 wt% after ten years, rather than 3.65 wt%. The associated temperature-swing formula returns 26.982108 K rather than 14 K at ten years. Both asymptotes are fitted scenario assumptions, not predicted ecosystem outcomes.

Image 157 describes thermal penetration depth for periodic conduction; it cannot alone prove a change in surface forcing amplitude. Image 164 must be corrected to $P = E - \partial_t W - \text{div}(F_q)$ when F_q is the outward moisture flux. If a convergence variable is used instead, it must be defined as minus divergence. Source percentages such as 46.8% recycling, 215 mm/year additional rainfall and 14.2% demand reduction are unverified model assumptions, not consequences of the conservation equation.

12. Explicit derivations and reproducible benchmark problems

Galerkin error: subtract the continuous and discrete weak equations to obtain $a(u - u_h, v_h) = 0$. For any v_h in V_h , coercivity and orthogonality give $\alpha \text{norm}(e)^2 \leq a(e, e) = a(e, u - v_h) \leq M \text{norm}(e) \text{norm}(u - v_h)$. Dividing by $\text{norm}(e)$, or handling $e = 0$ separately, gives M_06 . This proof does not cover noncoercive mixed systems without an appropriate stability argument.

Heat benchmark: prescribe a rod $0 < x < L$, constant thermal diffusivity α , zero excess temperature at both ends and initial $\theta(x, 0) = \sin(\pi x/L)$. Two x derivatives give $\theta_{xx} = -(\pi/L)^2 \theta$; one time derivative gives $\theta_t = -\alpha(\pi/L)^2 \theta$, proving P32. The explicit scheme E49 is stable for $Fo \leq 1/2$. The implementation chooses $Fo \leq 0.4$ and reports actual dt , number of steps and maximum pointwise error against the exact solution. The temperatures are normalized; this is a solver verification case, not a material experiment.

Quantum box: for $V = 0$ inside and infinite walls, solve $\psi'' + k^2 \psi = 0$. $\psi(0) = \psi(L) = 0$ gives $k = n \pi/L$. Normalization gives $\sqrt{2/L}$, and $E = \hbar^2 k^2 / (2m)$ gives P19. A finite-difference eigenvalue reference is $E_{n,h} = 2 \hbar^2 \sin^2(n \pi / [2(N+1)]) / (m \, dx^2)$, $dx = L/(N+1)$, for N interior points. It converges to the continuum energy as N increases. This exact discrete spectrum is a benchmark for classical numerical solvers, not a QPU materials calculation.

Canal and tunnel: continuity imposes one total Q through the series elements, while velocities differ by area. Insert $v_t = Q/At$ into Darcy-Weisbach and $v_o = Q/A_o$ into Manning's friction slope, add losses to ΔH , and solve the resulting positive scalar quadratic in Q . This proves E39 for fixed cross-sections and the chosen minor-loss convention. Real free-surface backwater, gates and transients require a more complete model.

Soil carbon: integrating $dC/dt + kC = I$ with constant k and I gives $C(t) = I/k + (C_0 - I/k) \exp(-kt)$. An observed equilibrium and an assumed timescale are inputs; the formula cannot predict both without data. Multi-pool extensions replace scalar k with a transfer/decomposition matrix and must conserve carbon through explicitly accounted respiration, exports and inputs.

Thin-wall cylinder: equilibrium of a longitudinal half-cylinder gives $2 \sigma_{\theta} t L = p D L$, hence E11. It neglects radial variation and local discontinuities. A laminate pipe must resolve ply stresses, membrane/bending coupling, defects, joints and environmental degradation; using bare fibre strength as pipe allowable is invalid.

Reliability bound: if all n independent trials succeed, $\Pr(\text{all success} | p) = p^n$. Setting $p^n = \alpha$ gives the one-sided lower endpoint $p_{\text{lower}} = \alpha^{1/n}$. This derivation makes the data requirement explicit and does not establish a probability for an undefined concept such as total project feasibility.

13. Constants, units and data provenance

Use exact SI defining values $c=299792458$ m/s, $h=6.62607015e-34$ J s, $e=1.602176634e-19$ C, $k_B=1.380649e-23$ J/K and $N_A=6.02214076e23$ 1/mol. Set $\hbar=h/(2\pi)$ and $R=N_A k_B$. The standard gravity value 9.80665 m/s² is a conventional reference, not the local gravity at every site. Electron mass, gravitational constant, vacuum permeability and material parameters are measured quantities with uncertainties; do not silently classify them as exact [BIPM].

Each parameter record requires name, symbol, SI value, display unit, uncertainty or interval, source, revision/date, measurement method, environmental conditions, material/lot and allowed range. Avoid overloaded names across coupled modules: distinguish mass density ρ_m , electrical resistivity ρ_e , reflectance ρ_λ , dynamic viscosity μ_v and shear modulus μ_s . Kelvin and Celsius increments coincide but absolute thermodynamic temperature must use kelvin. Activation energies per mole use R; per-particle energies use k_B .

The job manifest records all transforms and conversions. Reject undefined units, NaN, infinity, negative dimensions, zero efficiency, recovery outside (0,1), unsupported material regimes and missing critical coefficients. For a timestep or mesh choice report the mathematical stability condition and the accuracy study separately. Record numerical precision and nondeterministic backend behaviour rather than promising bitwise agreement across CPU/GPU/QPU hardware.

14. Product-family dossier and Fab routing specification

The package includes provisional dossier PDFs for eight source-derived component families plus the simple cylinder demonstration. The registry is intentionally bounded and not a claim to enumerate every possible product in the book. Every dossier separates source dimensions, engineering unknowns, candidate process route, formula references, cost inputs, patent-review status and EU applicability questions. Costs and patent clearance remain unknown where evidence is missing; zero is not substituted for missing information.

The manufacturing graph supports multiple visits to a Fab. A composite pressure pipe may require Fab 1 coating research, Fab 2 instrument calibration, Fab 3 mandrels/end fittings, Fab 4 layup/cure and Fab 5 proof testing and assembly. A metallic bracket may route directly through Fab 3 and Fab 5. These are workflow hypotheses until material/process capability and inspection records prove suitability.

A real product release requires toleranced geometry and interfaces, requirements traceability, a failure-mode analysis appropriate to the product, validated material allowables, loads and life, tool-access/collision simulation, fixtures, calibrated machines and inspection, first-article data and controlled revisions. For a nonprintable feature, evaluate machining access, casting cores, joining and modular redesign. If no qualified process exists, return UNSUPPORTED, not an invented machine or a simulated success.

The prototype exports a cylinder surface mesh in millimetres. Its radial chord error is bounded by E56, allowing the chosen tessellation to be checked against a drawing requirement. STL does not preserve analytic cylinders, feature history, tolerances or material semantics. Production exchange should use validated CAD assemblies and controlled drawings; a future STEP AP242 adapter needs a real geometry kernel and round-trip tests.

15. Extended software contracts and platform acceptance

Reference layers: (1) typed units and input validation, (2) model registry with applicability, (3) deterministic job orchestration and provenance, (4) solver adapters, (5) uncertainty and verification, (6) product/CAD registry, (7) process-capability and CAM planning, (8) metrology results and release controls, (9) multilingual presentation and export. Each adapter reports NOT_CONFIGURED, INVALID_INPUT, RUNNING, FAILED, CONVERGED or VALIDATED_FOR_SCOPE; convergence and validation are distinct states.

Classical CPU solvers remain the reference for tractable benchmark cases. A QPU adapter must declare Hamiltonian mapping, ansatz or algorithm, compilation target, backend calibration, shots, mitigation, statistical uncertainty, resource estimate and classical comparison. The provided Bell-state OpenQASM example only illustrates syntax; it is not a software port of continuum mechanics to a quantum computer. No provider job is executed in this package.

Windows, macOS, Linux and Android require separate packaging, dependency, accessibility, locale, file-permission, export and performance tests. A responsive HTML application provides a common interface, but does not establish native APIs or real-time scheduling. Apple Metal, DirectX, Vulkan and PREEMPT_RT remain architecture options; none is claimed as implemented by the browser prototype. Safety-related machine control needs a separate validated control system and commissioning process.

Virtual resolution is a display resource setting. At 4K, 3840 x 2160 RGBA8 requires approximately 33.18 MB for one frame buffer; RGBA16F requires approximately 66.36 MB. Increasing each dimension by 1000 multiplies those figures by one million before depth, history and simulation buffers. Demand-driven tiles and level of detail manage storage, but do not create a corresponding physical holographic instrument. The source's phrase 32-bit RGBA FP16 is inconsistent if it means 16 bits per each of four channels; RGBA16F uses 64 bits per pixel.

16. Test evidence and remaining decision gates

The software tests cover the corrected irrigation arithmetic, linear scaling with area, laminar limit, invalid input rejection, zero-failure confidence threshold, cylinder mesh topology and volume, cost arithmetic, conservation of head across the canal/tunnel screening and heat-equation convergence. Browser tests cover all six language switches, calculations, invalid inputs, JSON save/load, STL download, responsive layout and uncaught errors. Symbolic tests cover selected elasticity representations. These tests validate those code paths only.

The source document's legacy programs contain hard-coded outcomes, incomplete formatting and unqualified success flags. They remain historical source listings; translating comments or adding a review does not make them production-ready. The delivered core is separate, small and reproducible. The source equations, narrative and code may describe inconsistent scenarios; do not combine their totals unless a scenario manifest proves that they share inputs.

Open gates are the complete intended source edition and Engineering PDF; independent technical editorial review; real product and machine data; full norm texts and current applicability; patent claims/status review; supplier quotations; site and material measurements; calibrated multiphysics solvers; hardware integration; manufacturing trials and independent acceptance. No number of generated pages substitutes for those inputs and tests.

Mathematics — formula register

M01 · Scale coordinate

$$L = 10^k \text{ m}; k = \log_{10}(L / 1 \text{ m})$$

Assumptions and units: L is a positive physical length in metres; k is dimensionless. A view coordinate does not prescribe mesh spacing or instrument accuracy.

Verification: Check units and positivity; choose governing model independently. [BIPM]

M02 · Nondimensionalization

$$x^* = x/L_0; t^* = t/t_0; u^* = u/U_0$$

Assumptions and units: L_0 , t_0 and U_0 are positive reference scales with the units of their variables.

Verification: Report reference values with every dimensionless tolerance. [BIPM]

M03 · Linear system and forward error

$$A x = b; r = b - A x_h; \|x - x_h\| \leq \|A^{-1}\| \|r\|$$

Assumptions and units: A is nonsingular; compatible induced norm. x_h is an approximation. Residual alone does not bound error without stability information.

Verification: Estimate conditioning and verify an independent residual. [NUM]

M04 · Newton iteration

$$J(x_k) \delta = -F(x_k); x(k+1) = x_k + \alpha \delta$$

Assumptions and units: F differentiable; local nonsingular Jacobian J ; α in $(0, 1]$. Global convergence is not automatic.

Verification: Check scaled residual, step, line search and domain constraints. [NUM]

M05 · Weak Poisson problem

$$\text{Integral}_{\Omega} k \text{ grad}(u) \cdot \text{grad}(v) \, dV = \text{Integral}_{\Omega} f v \, dV + \text{Integral}_{\Gamma_N} g v \, dS$$

Assumptions and units: Solve $-\text{div}(k \text{ grad } u) = f$; u prescribed on Γ_D ; $k \text{ grad}(u) \cdot n = g$ on Γ_N ; k bounded and uniformly positive.

Verification: Coercivity with appropriate Dirichlet boundary; pure Neumann needs compatibility and a fixed mean. [FEM]

M06 · Galerkin quasi-optimality

$$a(u - u_h, v_h) = 0; \|u - u_h\|_V \leq (M/\alpha) \inf_{v_h} \|u - v_h\|_V$$

Assumptions and units: Continuous bilinear form a with continuity M and coercivity $\alpha > 0$; conforming $V_h \subset V$.

Verification: Applies to this coercive setting, not every nonlinear or saddle-point model. [FEM]

M07 · Residual estimator

$$\eta^2 = \sum_K h_K^2 \|f + \operatorname{div}(k \operatorname{grad} u_h)\|_K^2 + \sum_e h_e \|[\![\operatorname{jump}(k \operatorname{grad} u_h.n)]\!] \|_e^2$$

Assumptions and units: Illustrative diffusion estimator with matching boundary residuals; coefficient jumps require weighting. Norms are L^2 .

Verification: Reliability constants, data oscillation and regularity must be documented. [FEM]

M08 · Richardson order estimate

$$p = \log(|(q_3 - q_2)/(q_2 - q_1)|) / \log(r)$$

Assumptions and units: $h_3 = r h_2 = r^2 h_1$, $r > 1$; asymptotic leading error $q(h) = q_{\text{exact}} + C h^p$ and nonzero differences.

Verification: Use at least three refinements; do not use near cancellation or outside asymptotic range. [NUM]

M09 · Mixed-method stability

$$\inf_{q_h} \sup_{v_h} b(v_h, q_h) / (\|v_h\|_V \|q_h\|_Q) \geq \beta_0 > 0$$

Assumptions and units: Incompressible mixed FEM needs an inf-sup stable pair on the actual mesh family, or consistent stabilization.

Verification: Taylor-Hood is a method choice, not a universal proof for all geometries. [FEM]

M10 · Uncertainty propagation

$$\operatorname{uc}^2(y) = \sum_i \sum_j (df/dx_i)(df/dx_j) \operatorname{cov}(x_i, x_j)$$

Assumptions and units: First-order propagation for $y = f(x)$; include covariance and measurement model inputs.

Verification: For strong nonlinearity propagate distributions and record sampling settings. [NIST]

M11 · Zero-failure reliability bound

$$p_{\text{lower}} = \alpha^{1/n}; n \geq \lceil \log(\alpha) / \log(p_{\text{target}}) \rceil$$

Assumptions and units: n independent representative Bernoulli trials, all successful; one-sided confidence $1 - \alpha$. For $p_{\text{target}} = 0.99$ and $\alpha = 0.05$, $n = 299$.

Verification: A lower bound on one defined pass probability is not total engineering feasibility. [DERIVED]

M12 · Bounded integer search

$$D(x_1, \dots, x_n) = 0; x_i \in \{-M, \dots, M\}; \text{cases} = (2M+1)^n$$

Assumptions and units: Finite M and integer polynomial D ; exhaustive finite search is decidable.

Verification: Timeout means unknown; finite absence does not prove unbounded absence. [NUM]

M13 · Constrained optimization

$\min_x f(x)$, subject to $g(x) \leq 0$ and $h(x) = 0$

Assumptions and units: Geometry, manufacturing and reliability constraints must be explicit.

Verification: Local stationary points are not global certificates for nonconvex problems. [NUM]

M14 · Linear reduced model

u approximately $V a$; $(V^T A V) a = V^T b$

Assumptions and units: Columns of V form a selected reduced basis; A and b refer to a discretized problem.

Verification: Validate outside training samples and retain a residual-based estimator. [FEM]

M15 · Lax-Milgram existence statement

$a(u, v) = l(v)$ for all v in V ; $\text{norm}(u) \leq \text{norm}(l)/\alpha$

Assumptions and units: Real Hilbert space V ; bounded coercive bilinear a with coercivity $\alpha > 0$; continuous linear functional l .

Verification: Existence and uniqueness under these hypotheses; not a statement about arbitrary PDEs. [FEM]

M16 · Gronwall bound

$y(t) \leq a + \int_0^t b(s)y(s)ds$ implies $y(t) \leq a \exp(\int_0^t b(s)ds)$

Assumptions and units: $a \geq 0$; y, b nonnegative and appropriately integrable.

Verification: A priori amplification estimate; a large bound is not a useful numerical error certificate. [NUM]

M17 · Contraction iteration

$\text{norm}(Tx - Ty) \leq q \text{norm}(x - y)$; $\text{norm}(x^* - x_k) \leq q^k \text{norm}(x_1 - x_0)/(1 - q)$

Assumptions and units: $0 \leq q < 1$; complete metric/normed setting and invariant iteration set.

Verification: Prove contraction on the actual domain before asserting convergence. [NUM]

M18 · SVD regularization

$A = U \Sigma V^T$; $x_\lambda = \sum_i [\sigma_i / (\sigma_i^2 + \lambda^2)] (u_i^T b) v_i$

Assumptions and units: Tikhonov penalty $\lambda^2 \text{norm}(x)^2$ with $\lambda > 0$, Euclidean norms.

Verification: Regularization introduces bias; choose λ by data uncertainty and validation. [NUM]

M19 · Preconditioned conjugate gradients

$$A x=b; \text{error_A}(k) \leq 2[(\sqrt{\kappa}-1)/(\sqrt{\kappa}+1)]^k \text{error_A}(0)$$

Assumptions and units: Symmetric positive definite A and suitable SPD preconditioner; kappa of preconditioned operator.

Verification: Exact-arithmetic bound; monitor finite-precision residual and actual convergence. [NUM]

M20 · Generalized eigenproblem

$$K \phi = \omega^2 M \phi; \phi_i^T M \phi_j = \delta_{ij}$$

Assumptions and units: Symmetric stiffness K, SPD mass M, conservative linear vibration problem.

Verification: Constraints remove rigid modes as intended; damping and gyroscopic effects alter formulation. [FEM]

M21 · Adjoint sensitivity

$$R(u,p)=0; R_u^T \lambda = J_u^T; dJ/dp = J_p - \lambda^T R_p$$

Assumptions and units: Differentiable residual and objective with locally invertible state Jacobian.

Verification: Check against finite differences; nonsmooth contact or topology changes need special handling. [NUM]

M22 · Interval Newton enclosure

$$N(X) = m - [F'(X)]^{-1} F(m)$$

Assumptions and units: Interval Jacobian over box X, point m in X; an appropriate interval linear solver is needed.

Verification: Inclusion and uniqueness conditions are method-specific; interval overestimation can prevent certification. [NUM]

M23 · Bayesian calibration

$$p(\theta|d) \text{ proportional to } p(d|\theta) p(\theta)$$

Assumptions and units: Specify prior, likelihood, noise and model discrepancy; posterior is conditional on these choices.

Verification: Do not equate posterior confidence with externally validated physical truth. [NUM]

M24 · Monte Carlo standard error

$$SE(\text{mean}) = s/\sqrt{N}$$

Assumptions and units: Independent identically distributed samples with finite variance; correlated chains need effective sample size.

Verification: Record seeds and convergence; deterministic reproducibility does not eliminate sampling uncertainty. [NUM]

M25 · Gaussian chance constraint

$$\Pr(g \leq 0) \geq 1 - \alpha \text{ is equivalent to } \mu_g + z_{1-\alpha} \sigma_g \leq 0$$

Assumptions and units: Exactly Gaussian scalar g with known mean/standard deviation; z is standard-normal quantile.

Verification: Approximate for non-Gaussian nonlinear responses; include parameter estimation uncertainty. [NUM]

M26 · Topology-optimization interpolation

$$E(\rho) = E_{\min} + \rho^p (E_0 - E_{\min}); 0 \leq \rho \leq 1$$

Assumptions and units: SIMP interpolation, penalization $p > 1$; positive $E_{\min}$; density-filter/projection definition required.

Verification: Mesh dependence, minimum feature, buckling, fatigue and manufacturing constraints need explicit treatment. [FEM]

M27 · Discrete conservation at interfaces

$$\sum_{\text{cells}} \Delta \text{ conserved_quantity} + \text{Integral boundary_flux } dt = \text{sources}$$

Assumptions and units: Every internal face uses equal and opposite fluxes in a common unit convention.

Verification: Check global and local budgets; interpolation between nonmatching meshes must preserve transfer quantities. [DERIVED]

M28 · CFL condition for upwind transport

$$C = \text{abs}(a) \, dt/dx \leq 1$$

Assumptions and units: 1D linear advection, uniform grid, explicit first-order upwind scheme.

Verification: This bound is scheme-specific; not a universal timestep criterion for all PDE solvers. [NUM]

M29 · Energy-stable backward Euler diffusion

$$(\text{norm}(u_{\text{new}})^2 - \text{norm}(u_{\text{old}})^2) / (2dt) + k \, \text{norm}(\text{grad } u_{\text{new}})^2 \leq 0$$

Assumptions and units: Homogeneous diffusion, zero source, compatible homogeneous boundaries, $k > 0$ and suitable inner product.

Verification: Unconditional linear stability is not unconditional time accuracy. [FEM]

M30 · Dimensionless scalar error budget

$$e_{\text{total}} \leq e_{\text{discretization}} + e_{\text{iteration}} + e_{\text{input}} + e_{\text{model}}$$

Assumptions and units: Triangle-inequality bound only if every term bounds the same quantity in compatible units/norms.

Verification: Unknown model discrepancy must remain unknown; do not assign it a convenient zero. [DERIVED]

Physics — formula register

P01 · Mass conservation

$$\partial_t \rho + \operatorname{div}(\rho \mathbf{v}) = s_m$$

Assumptions and units: ρ kg/m³; velocity $\mathbf{v}$ m/s; source s_m kg/(m³ s).

Verification: Integrate flux and storage independently at all scale interfaces. [CONT]

P02 · Incompressible momentum

$$\rho(\partial_t \mathbf{v} + (\mathbf{v} \cdot \operatorname{grad}) \mathbf{v}) = -\operatorname{grad} p + \mu \operatorname{laplacian} \mathbf{v} + \mathbf{f}; \operatorname{div} \mathbf{v} = 0$$

Assumptions and units: Newtonian constant-density fluid; μ Pa s; $\mathbf{f}$ force per volume N/m³. Specify initial and boundary data.

Verification: Monitor momentum residual and divergence; validate mesh/time refinement. [CONT]

P03 · Heat transport

$$\rho c_p(\partial_t T + \mathbf{v} \cdot \operatorname{grad} T) = \operatorname{div}(\mathbf{k} \operatorname{grad} T) + Q$$

Assumptions and units: T K; c_p J/(kg K); $\mathbf{k}$ W/(m K); Q W/m³. Constant-property sensible heat approximation.

Verification: Include latent heat, radiation and thermo-mechanical work when material conditions require. [CONT]

P04 · Closed-system first law

$$\Delta E = Q_{\text{in}} - W_{\text{out}}$$

Assumptions and units: E J includes relevant internal, kinetic and potential energy; heat Q and work W signed as stated.

Verification: Energy conservation checks the balance residual, not whether ΔE is near zero. [THERMO]

P05 · Entropy balance

$$\Delta S = \int \delta Q/T_{\text{boundary}} + S_{\text{gen}}; S_{\text{gen}} \geq 0$$

Assumptions and units: Closed system may exchange heat; isolated system has zero heat and mass exchange.

Verification: A closed system can lose entropy; test production including transfers. [THERMO]

P06 · Scaled balance residual

$$rE = |\Delta E - Q_{\text{in}} + W_{\text{out}}| / \max(E_{\text{ref}}, |\Delta E|, |Q_{\text{in}}|, |W_{\text{out}}|)$$

Assumptions and units: $E_{\text{ref}} > 0$ J chosen and reported; rE dimensionless.

Verification: Tolerance derived from discretization and metrology; 1e-12 J is not universal. [DERIVED]

P07 · Small-strain elasticity

$$\epsilon = (\text{grad } u + \text{grad } u^T)/2; \sigma = \lambda \text{tr}(\epsilon) I + 2 \mu \epsilon$$

Assumptions and units: u m; stress Pa; isotropic linear elastic solid; infinitesimal strain.

Verification: For stable ordinary compressible material $E > 0$ and $-1 < \nu < 0.5$. [CONT]

P08 · Lamé parameters and equilibrium

$$\mu = E/[2(1+\nu)]; \lambda = E \nu/[(1+\nu)(1-2\nu)]; \mu \text{laplacian } u + (\lambda + \mu) \text{grad div } u + f = 0$$

Assumptions and units: E Pa; ν dimensionless; f N/m³; homogeneous coefficients.

Verification: Near incompressibility needs suitable mixed methods; boundary data remain essential. [CONT]

P09 · Papkovitch-Neuber, force-free form

$$u = B - \text{grad}(r \cdot B + \phi)/[4(1-\nu)]; \text{laplacian } B = 0; \text{laplacian } \phi = 0$$

Assumptions and units: Homogeneous isotropic elasticity with zero body force; harmonic potentials; r m, B m, ϕ m².

Verification: Verified operator identity; solve boundary conditions. Nonzero body force needs a valid particular solution. [DERIVED]

P10 · Many-electron Hamiltonian

$$H = -\sum_i \hbar^2 \text{laplacian}_i / (2 m_e) - \sum_{iA} \frac{e^2}{4 \pi \epsilon_0 |r_i - R_A|} + \sum_{i < j} \frac{e^2}{4 \pi \epsilon_0 |r_i - r_j|} + E_{\text{NN}} | \text{Fixed nuclei, nonrelativistic Coulomb model; } E_{\text{NN}} = \sum_{A < B} \frac{Z_A Z_B e^2}{4 \pi \epsilon_0 |R_A - R_B|}$$

Assumptions and units:); antisymmetric electronic states.

Verification: No universal $N \geq 3$ impossibility threshold; finite-basis diagonalization is possible but can scale prohibitively. [QM]

P11 · Kohn-Sham equations

$$[-\hbar^2 \text{laplacian} / (2 m_e) + v_{\text{ext}} + v_H + v_{\text{xc}}] \psi_i = \epsilon_i \psi_i; n = \sum_i f_i |\psi_i|^2$$

Assumptions and units: f_i occupation; n electrons/m³; normalization $\int n \, dV = N_e$.

Verification: Converge basis, k mesh and SCF; PBE does not provide a universal bounded model error. [QE]

P12 · Kohn-Sham energy

$$E[n] = T_s[n] + \int v_{\text{ext}} n \, dV + E_H[n] + E_{\text{xc}}[n] + E_{\text{NN}}$$

Assumptions and units: Electronic ground-state functional with nuclear contribution for total energy; convention must be consistent.

Verification: Converged electronic numerics do not prove agreement with experimental properties. [QE]

P13 · Velocity Verlet

$$r_{\text{new}} = r + v \, dt + \frac{1}{2} a \, dt^2; \quad v_{\text{new}} = v + (a + a_{\text{new}}) dt/2$$

Assumptions and units: $a = F/m$ evaluated from chosen force model before and after position update.

Verification: Time-step refinement, energy drift and force-model validation required. [MD]

P14 · Pair-potential virial stress

$$\sigma = -(\sum_i m_i c_i \text{ tensor } c_i + 0.5 \sum_i \sum_{j \neq i} r_{ij} \text{ tensor } f_{ij})/V$$

Assumptions and units: Tension-positive convention; c_i peculiar velocity relative to local mean; r_{ij} points j to i ; f_{ij} force on i from j ; pair forces.

Verification: Pressure tensor has opposite sign; many-body potentials require appropriate virial decomposition. [MD]

P15 · Continuum validity indicators

$$Kn = \lambda_{\text{mfp}}/L; \quad Re = \rho U L/\mu; \quad Ma = U/c_{\text{sound}}$$

Assumptions and units: Mean free path λ_{mfp} and characteristic length L in m.

Verification: Select slip, kinetic or compressible models by regime; scale alone is insufficient. [CONT]

P16 · Maxwell equations

$$\text{div } D = \rho_e; \quad \text{div } B = 0; \quad \text{curl } E = -\partial_t B; \quad \text{curl } H = J + \partial_t D$$

Assumptions and units: SI fields plus constitutive relations, e.g. $D = \epsilon E$, $B = \mu H$ where applicable.

Verification: Frequency-dependent, anisotropic or nonlinear media need corresponding constitutive laws. [EM]

P17 · Diffraction-limited imaging

$$d \text{ approximately } \lambda/(2 \text{ NA})$$

Assumptions and units: Lateral Abbe scale under the stated imaging assumptions; wavelength and NA matter.

Verification: Virtual pixels do not establish molecular optical resolution or fabrication capability. [EM]

P18 · Ideal gas density

$$\rho = p M/(R T)$$

Assumptions and units: p Pa; molar mass M kg/mol; R J/(mol K); T K. R is universal, not the specific gas constant in this expression.

Verification: Use real-gas equations where required; local weather and gas temperature are inputs. [THERMO]

P19 · One-dimensional quantum box

$$E_n = n^2 \pi^2 \hbar^2 / (2m L^2); \psi_n = \sqrt{2/L} \sin(n \pi x/L)$$

Assumptions and units: Infinite potential walls, $0 < x < L$, n positive integer, nonrelativistic single particle.

Verification: An exact benchmark, not an electronic-structure model of an arbitrary product. [QM]

P20 · Harmonic oscillator spectrum

$$E_n = \hbar \omega (n + 1/2)$$

Assumptions and units: One-dimensional quadratic potential $V = m \omega^2 x^2 / 2$, $\omega > 0$, $n \geq 0$.

Verification: Real molecular vibrations require force constants and anharmonic/coupled-mode corrections. [QM]

P21 · Born-Oppenheimer force

$$F_A = -\partial E_{\text{electronic_total}} / \partial R_A$$

Assumptions and units: Adiabatic ground-state potential-energy surface, consistent total energy.

Verification: Incomplete basis may need Pulay corrections; nonadiabatic transitions need additional dynamics. [QE]

P22 · Transition-state rate estimate

$$k(T) = \kappa(T) (k_B T/h) \exp(-\Delta G_{\ddagger} / (R T))$$

Assumptions and units: Molar activation free energy J/mol; transmission coefficient κ ; appropriate standard-state conventions.

Verification: Recrossing, tunnelling and multiple pathways require treatment; activation data cannot be invented. [THERMO]

P23 · Boltzmann distribution

$$p_i = \exp(-E_i / (k_B T)) / Z; Z = \sum_i \exp(-E_i / (k_B T))$$

Assumptions and units: Canonical equilibrium, states and degeneracy specified; $T > 0$.

Verification: Non-equilibrium processes and strongly driven manufacturing are outside this equilibrium statement. [THERMO]

P24 · Fluctuation-dissipation Langevin model

$$m dv = -\gamma v dt + F dt + \sqrt{2 \gamma k_B T} dW$$

Assumptions and units: Markovian white-noise bath, γ kg/s, W Wiener process; chosen stochastic convention.

Verification: Validate thermostat and time integration; seed reproducibility is not independent replication. [MD]

P25 · Boltzmann transport

$$\partial_t f + \mathbf{v} \cdot \nabla_x f + \mathbf{a} \cdot \nabla_v f = C[f]$$

Assumptions and units: Distribution f in phase space; collision operator requires a physical model.

Verification: For rarefied gases choose collision cross sections, boundaries and statistical error controls. [CONT]

P26 · Radiative transfer

$$dI_\lambda/ds = -\kappa_\lambda I_\lambda + j_\lambda$$

Assumptions and units: No scattering in this simplified ray form; intensity, absorption and emission use consistent spectral units.

Verification: Scattering, wavelength integration and view geometry must be added when relevant. [EM]

P27 · Thermoelastic coupling

$$\sigma = C:(\epsilon - \epsilon_{th}); \rho u_{tt} = \text{div } \sigma + f$$

Assumptions and units: Small-strain elastic mechanics with thermal eigenstrain; C units Pa.

Verification: Temperature-dependent properties, plastic dissipation and geometric nonlinearities require extension. [CONT]

P28 · General finite-strain balances

$$\mathbf{F} = \mathbf{I} + \text{Grad } \mathbf{u}; J = \det \mathbf{F} > 0; \rho_0 = \rho J; \text{Div } \mathbf{P} + \rho_0 \mathbf{b} = \rho_0 u_{tt}$$

Assumptions and units: Reference-coordinate deformation gradient $\mathbf{F}$, first Piola stress $\mathbf{P}$, mass density ρ_0 .

Verification: Constitutive law must be objective and thermodynamically admissible; prevent inverted elements. [CONT]

P29 · Newtonian gravitation and field

$$\nabla^2 \Phi = 4\pi G \rho; \mathbf{a} = -\nabla \Phi$$

Assumptions and units: Weak-field slow-motion approximation; boundary conditions and mass distribution specified.

Verification: A scale slider cannot extend Newtonian validity into relativistic regimes. [GRAV]

P30 · Einstein field equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4) T_{\mu\nu}$$

Assumptions and units: Metric-sign and curvature conventions specified consistently; stress-energy tensor conserved covariantly.

Verification: Included as a domain boundary, not a solved cosmological manufacturing engine. [GRAV]

P31 · Friedmann expansion equation

$$H^2 = (8\pi G/3)\rho + \Lambda c^2/3 - k c^2/a^2$$

Assumptions and units: Homogeneous isotropic cosmology; ρ mass-equivalent density; choose a in length units and k dimensionless.

Verification: Requires cosmological parameters and assumptions; unrelated to increasing machine resolution. [GRAV]

P32 · Thermal diffusion analytic benchmark

$$\theta(x,t) = \sin(\pi x/L) \exp(-\alpha \pi^2 t/L^2)$$

Assumptions and units: 1D rod, fixed zero excess temperature at endpoints, initial unit sine, constant $\alpha > 0$.

Verification: Substitute in $\theta_t = \alpha \theta_{xx}$; compare numerical solution with exact profile. [DERIVED]

Materials — formula register

S01 · Fick transport

$$J = -D \operatorname{grad} c; \operatorname{partial}_t c = \operatorname{div}(D \operatorname{grad} c) + s$$

Assumptions and units: c mol/m³; D m²/s; J mol/(m² s); s mol/(m³ s).

Verification: State phase, temperature, concentration dependence and interface conditions. [MAT]

S02 · Arrhenius kinetics

$$D = D_0 \exp(-E_a/(R T))$$

Assumptions and units: E_a J/mol and R J/(mol K); use $E_a/(k_B T)$ only for energy per particle.

Verification: Fit D_0, E_a with uncertainty; do not mix molar and particle activation energies. [MAT]

S03 · Allen-Cahn evolution

$$F[c] = \int (f(c) + \kappa |\operatorname{grad} c|^2/2) dV; \operatorname{partial}_t c = -L \operatorname{delta} F/\operatorname{delta} c$$

Assumptions and units: Nonconserved order parameter, $L \geq 0$, suitable boundaries.

Verification: Check nonincreasing free energy without external work; coefficients need calibration. [MAT]

S04 · Cahn-Hilliard evolution

$$\mu_c = df/dc - \kappa \operatorname{laplacian} c; \operatorname{partial}_t c = \operatorname{div}(M \operatorname{grad} \mu_c)$$

Assumptions and units: Conserved field c , mobility $M \geq 0$; no-flux closed domain conserves integral c .

Verification: Check mass conservation and energy decay; boundary conditions on c and μ_c . [MAT]

S05 · Linear homogenization

$$C_{\text{eff}} = \text{volume_average}(C(x):A(x)); \epsilon(x) = A(x):\epsilon_{\text{macro}}$$

Assumptions and units: Representative volume element and scale separation; A is strain localization tensor.

Verification: Converge RVE size and ensure macro-micro work consistency. [MAT]

S06 · Hill-Mandel condition

$$\text{average}(\sigma:\operatorname{delta} \epsilon) = \sigma_{\text{macro}}:\operatorname{delta} \epsilon_{\text{macro}}$$

Assumptions and units: Work equivalence under admissible micro-boundary conditions.

Verification: Track interface tractions and displacements; simple parameter copying is insufficient. [MAT]

S07 · Thermal strain

$\epsilon_{th} = \int_{T_{ref}}^T \alpha(\theta) d\theta$ | Scalar for isotropic thermal expansion; tensor form for anisotropy. | Constrained uniaxial elastic magnitude E

Assumptions and units: ϵ_{th}

Verification: is a special case, not universal stress. [MAT]

S08 · J2 yield criterion

$\sigma_{eq} = \sqrt{3 s : s / 2}$; $f = \sigma_{eq} - \sigma_y \leq 0$

Assumptions and units: $s = \sigma - \text{tr}(\sigma)I/3$; isotropic ductile yield model; Pa.

Verification: Composite failure requires laminate allowables and appropriate failure criteria. [MAT]

S09 · Fracture mechanics

$KI = Y \sigma \sqrt{\pi a}$; require $KI < KIC$

Assumptions and units: Linear elastic mode I with crack size a , geometry factor Y and valid toughness KIC .

Verification: Small-scale yielding and specimen validity required; include environment and fatigue. [MAT]

S10 · Fatigue growth

$da/dN = C (\Delta K)^m$

Assumptions and units: Paris regime only; C and m are measured for material/environment/load ratio.

Verification: Exclude near-threshold and unstable-growth regimes from this approximation. [MAT]

S11 · Cure kinetics

$d\alpha/dt = (k_1 + k_2 \alpha^m)(1 - \alpha)^n$

Assumptions and units: Empirical autocatalytic model; $0 \leq \alpha \leq 1$; rates temperature-dependent.

Verification: Cure fraction alone does not prove zero creep or structural qualification. [MAT]

S12 · Pressure-driven membrane permeation

$J = (P_{perm}/t)(p_1 - p_2)$

Assumptions and units: Linear solution-diffusion regime; permeability units chosen to yield $\text{mol}/(\text{m}^2 \text{ s})$; t m.

Verification: Use measured species-specific law. Square-root pressure law is not universal for polymers or oxides. [MAT]

S13 · Chemical potential

$$\mu_i = (\partial G / \partial n_i)_{(T,p,n_j)}$$

Assumptions and units: Gibbs energy J, amount n_i mol; chemical potential J/mol.

Verification: Standard states and reference energies must match across phases. [THERMO]

S14 · Ideal mixing free energy

$$\Delta G_{\text{mix}} = R T \sum_i n_i \ln(x_i)$$

Assumptions and units: Ideal solution with mole fractions $x_i > 0$ and $\sum x_i = 1$.

Verification: Real alloys/electrolytes require excess Gibbs-energy/activity terms. [THERMO]

S15 · Phase equilibrium

$$T_{\alpha} = T_{\beta}; p_{\alpha} = p_{\beta}; \mu_{i\alpha} = \mu_{i\beta}$$

Assumptions and units: Equilibrium phases able to exchange heat, volume and relevant components.

Verification: Metastability and kinetic barriers may prevent equilibrium microstructure during manufacture. [MAT]

S16 · Classical nucleation

$$\Delta G(r) = 4 \pi r^2 \gamma + (4 \pi / 3) r^3 \Delta g_v; r^* = -2 \gamma / \Delta g_v$$

Assumptions and units: Spherical homogeneous nucleus, $\gamma > 0$ J/m², driving $\Delta g_v < 0$ J/m³.

Verification: Heterogeneous nucleation, anisotropy and atomistic scales can invalidate continuum capillarity. [MAT]

S17 · Critical nucleation barrier

$$\Delta G^* = 16 \pi \gamma^3 / [3(\Delta g_v)^2]$$

Assumptions and units: Same assumptions as S16; units J.

Verification: Nucleation rate also depends on a kinetic prefactor and site population. [MAT]

S18 · Diffusion length scaling

$$\ell_{\text{rms}} = \sqrt{2 d D t}$$

Assumptions and units: Isotropic Brownian diffusion in d dimensions, constant D and no boundaries/drift.

Verification: Not the same as a guaranteed penetration depth or interface thickness. [MAT]

S19 · Reaction-diffusion species balance

$$\partial_t c_i + \text{div}(c_i \mathbf{v}) = \text{div}(D_i \text{grad } c_i) + \sum_r \nu_{ir} R_r$$

Assumptions and units: Concentrations mol/m³, stoichiometric coefficients ν_{ir} , reaction rates mol/(m³ s).

Verification: Enforce elemental and charge conservation; electroneutral or Poisson coupling as appropriate. [MAT]

S20 · Nernst-Planck flux

$$J_i = -D_i \text{grad } c_i - z_i D_i F c_i \text{grad}(\phi)/(RT) + c_i v$$

Assumptions and units: Dilute isothermal ionic transport; z_i charge number, Faraday F C/mol.

Verification: Concentrated brines require nonideal electrochemical potentials and coupled transport. [GEOCHEM]

S21 · Faraday electrolysis balance

$$m = \eta_F M I t / (z F)$$

Assumptions and units: M kg/mol, I A, t s, z electrons per product unit, η_F current efficiency.

Verification: Mass yield does not include energy or separation cost; side reactions and purity need testing. [EM]

S22 · Butler-Volmer relation

$$j = j_0 [\exp(\alpha_a F \eta / (RT)) - \exp(-\alpha_c F \eta / (RT))]$$

Assumptions and units: Simple one-electron activation-controlled form, current densities A/m², overpotential η V.

Verification: Mass-transfer limitations, multistep kinetics, surface changes and concentrated electrolytes require extension. [GEOCHEM]

S23 · Crystal plasticity kinematics

$$F = F_e F_p; L_p = \text{dot}(F_p) \text{inverse}(F_p) = \sum_s \gamma_{\dot{s}} m_s \text{tensor } n_s$$

Assumptions and units: Slip directions/normals in the intermediate configuration; multiplicative finite-strain decomposition.

Verification: Slip systems, hardening and texture must be material-specific and calibrated. [MAT]

S24 · Rate-dependent slip rule

$$\gamma_{\dot{s}} = \gamma_0 \text{abs}(\tau_s / g_s)^n \text{sign}(\tau_s)$$

Assumptions and units: Empirical power-law slip, positive resistance g_s , rate γ_0 1/s.

Verification: Parameters depend on material, temperature and strain-rate regime; not universal. [MAT]

S25 · Orowan plastic shear rate

$$\gamma_{\dot{s}} = \rho_{\text{mobile}} b v_{\text{dislocation}}$$

Assumptions and units: Dislocation density 1/m², Burgers vector magnitude b , mean glide speed v m/s.

Verification: Needs slip-system resolution, mobile fraction and velocity law. [MAT]

S26 · Hall-Petch relation

$$\sigma_y = \sigma_0 + k_{HP} / \sqrt{d_{\text{grain}}}$$

Assumptions and units: Empirical grain-size strengthening law with calibrated k_{HP} and valid grain-size range.

Verification: Do not extrapolate to arbitrarily small grains; mechanisms and inverse trends may change. [MAT]

S27 · Norton creep law

$$\dot{\epsilon} = A \sigma^n \exp(-Q/(R T))$$

Assumptions and units: Empirical uniaxial steady-creep law; A units depend on stress units; Q J/mol.

Verification: Calibration domain, transient creep, rupture and multiaxiality must be addressed. [MAT]

S28 · Linear damage accumulation

$$D = \sum_i n_i / N_i$$

Assumptions and units: Miner rule for specified cycle classes and material fatigue data.

Verification: Load sequence, mean stress, environment and nonlinear damage can invalidate $D=1$ predictions. [MAT]

S29 · Moving heat-source balance

$$\rho c_p T_t = \text{div}(k \text{ grad } T) + q(x-vt) - \rho L_m \dot{f}_{\text{liquid}}$$

Assumptions and units: Simplified enthalpy balance with latent heat L_m J/kg and liquid fraction.

Verification: Laser absorption, melt convection, radiation and evaporation determine process adequacy. [MAT]

S30 · Thermal residual-stress screening

$$\sigma_{\text{thermal}} \text{ approximately } E \alpha \Delta T / (1-\nu)$$

Assumptions and units: Ideal isotropic equal biaxial in-plane restraint, plane stress and uniform cooling; sign by convention.

Verification: Not a universal welding/additive stress formula; actual constraints and plasticity matter. [MAT]

Engineering — formula register

E01 · Crop water demand

$$Q=A \text{ ETc}/[\text{eta_app} (1-\text{LF}) 86400]$$

Assumptions and units: A m²; ETc m/day; eta_app in (0,1]; LF in [0,1). Simplified steady model without effective rainfall or storage.

Verification: Subtract appropriate effective rain and soil-water contributions in a full balance. [FAO]

E02 · Crop coefficient

$$\text{ETc}=\text{Kc} \text{ ETo}$$

Assumptions and units: Kc dimensionless; ET in mm/day; standard non-water-stressed crop conditions.

Verification: Local weather, crop stage, soil and salinity are required; 32 C is not a universal optimum. [FAO]

E03 · FAO daily reference evapotranspiration

$$\text{ETo}=[0.408 \Delta(\text{Rn}-\text{G})+\text{gamma} (900/(\text{T}+273)) \text{u}^2(\text{es}-\text{ea})]/[\Delta+\text{gamma}(1+0.34 \text{u}^2)]$$

Assumptions and units: ETo mm/day; Rn,G MJ/(m² day); T C; u² m/s at 2 m; vapour pressures kPa; Delta,gamma kPa/C.

Verification: FAO reference grass assumptions and unit-specific coefficients; not arbitrary SI substitution. [FAO]

E04 · Pipe continuity

$$\text{Q_pipe}=\pi \text{ D}^2 \text{ v}/4; \text{N_parallel}=\text{ceil}(\text{Q_total}/\text{Q_pipe})$$

Assumptions and units: D inner diameter m; v mean velocity m/s; Q m³/s.

Verification: Parallel branches and route distribution must be solved as a hydraulic network. [CONT]

E05 · Swamee-Jain friction factor

$$\text{fD}=0.25/[\log_{10}(\text{epsilon}/(3.7\text{D})+5.74/\text{Re}^{0.9})]^2$$

Assumptions and units: Darcy friction factor for applicable turbulent pipe flow; roughness epsilon m.

Verification: For laminar fully developed round-pipe flow use 64/Re; transitional flow needs special treatment. [FLUID]

E06 · Head and pump input

$$\text{hf}=\text{fD} (\text{L}/\text{D}) \text{v}^2/(2\text{g}); \text{H}=\text{hf}+\Delta \text{z}+\sum \text{K} \text{v}^2/(2\text{g})+\Delta \text{p}/(\rho \text{g}); \text{P}=\rho \text{g} \text{Q} \text{H}/\text{eta}$$

Assumptions and units: H m; P W; eta overall pump efficiency. K minor-loss factors.

Verification: Elevation is endpoint energy difference; do not count every elevated section as net head. [FLUID]

E07 · Osmotic work approximation

$$w_{\min} = -(\pi_0/Y) \ln(1-Y)$$

Assumptions and units: π_0 Pa; recovery $0 < Y < 1$; $w_{\min}$ J/m³; idealized dilute solution and negligible salt permeation.

Verification: Real seawater requires activity-based thermodynamics; not a full plant SEC prediction. [THERMO]

E08 · Desalination mass balance

$$Q_f = Q_p/Y; Q_b = Q_f - Q_p; \text{cf } Q_f = c_p Q_p + c_b Q_b$$

Assumptions and units: Steady volumetric approximation with comparable densities; concentrations kg/m³.

Verification: Use mass flows with density corrections for accurate concentrate calculations. [DERIVED]

E09 · Specific-energy power conversion

$$P_{\text{GW}} = 0.0036 Q_{\text{m}^3\text{s}} \text{ SEC}_{\text{kWhm}^3}$$

Assumptions and units: SEC is electricity per permeate volume; Q is permeate flow.

Verification: Include pretreatment, intake, pumping, ZLD and distribution only once. [DERIVED]

E10 · Static buoyancy

$$F_{\text{net}} = (\rho_{\text{air}} - \rho_{\text{gas}})g V - m_{\text{payload}} g$$

Assumptions and units: Payload includes structure, contained fluid, envelope, connections and carried equipment.

Verification: Add tether, wind, thermal and dynamic loads; no certified design follows from static lift. [CONT]

E11 · Thin-wall hoop screening

$$\sigma_{\theta} = p D / (2t)$$

Assumptions and units: Thin, circular pressure wall; pressure p Pa; D, t m; ideal membrane screening.

Verification: Laminate anisotropy, joints, axial loads, defects and creep require further assessment. [CONT]

E12 · Small-sag cable approximation

$$H_0 \text{ approximately } w L^2 / (8 \delta)$$

Assumptions and units: Uniform transverse load w N/m referred to span, span L m, sag δ m; small sag.

Verification: Solve nonlinear catenary and combined loads for final design; wind and lift interact. [CONT]

E13 · Wind and vortex screening

$$F_{\text{per_length}} = 0.5 \rho C_d U^2 D; f_s = St U/D$$

Assumptions and units: C_d, St depend on shape and Reynolds number.

Verification: Comparing one natural frequency with one shedding frequency does not exclude flutter. [FLUID]

E14 · Water hammer screening

$$\Delta p = \rho a \Delta v$$

Assumptions and units: Rapid velocity change relative to wave transit; wave speed a depends on fluid and wall compliance.

Verification: Network transient solver needed for valve events, air pockets and pressure limits. [FLUID]

E15 · Salt and mineral ceiling

$$m_{\text{species_per_year}} = Q_f c_{\text{species}} \text{recovery_species seconds_year}$$

Assumptions and units: Q_f m³/s; concentration kg/m³; recovery in [0,1]; kg/year.

Verification: Stoichiometric ceiling only: purity, competing ions, separation energy and market demand still matter. [DERIVED]

E16 · Struvite stoichiometry



Assumptions and units: Moles product cannot exceed the limiting molar supply of Mg, N or P.

Verification: pH speciation, activities, kinetics and fertilizer demand must be quantified independently. [MAT]

E17 · Root-zone storage

$$dS/dt = P_{\text{eff}} + I - ET - R - D$$

Assumptions and units: All water terms as depth per time; storage S depth; R runoff; D drainage.

Verification: Water exported into atmosphere is not guaranteed to return to the same basin. [FAO]

E18 · Radiative and latent energy

$$q_{\text{rad}} = \epsilon \sigma_B (T_s^4 - T_a^4); q_{\text{lat}} = \dot{m} L_v$$

Assumptions and units: Radiative exchange approximation; q_{rad} W/m²; q_{lat} W if $\dot{m}$ kg/s.

Verification: Air mixing, humidity and advection bound evaporative cooling; visual mist proves no temperature reduction. [THERMO]

E19 · Cost per accepted product

$$C_{\text{unit}} = (C_{\text{material}} + C_{\text{machine}} + C_{\text{energy}} + C_{\text{labour}} + C_{\text{QA}} + C_{\text{overhead}}) / N_{\text{accepted}}$$

Assumptions and units: All costs in one currency and price date; $N_{\text{accepted}} > 0$; avoid double-counting energy in machine rates.

Verification: Use quotes, yield and ranges; no product price without geometry, material, quantity and process. [DERIVED]

E20 · Levelized water cost

$$\text{LCOW} = \frac{\sum_t C_t / (1+r)^t}{\sum_t V_t / (1+r)^t}$$

Assumptions and units: Costs C_t include investment, operations and retirement net of stated residual value; V_t sold water; discount r .

Verification: Specify currency, base year, real/nominal consistency and financing assumptions. [DERIVED]

E21 · Process capability

$$C_p = (\text{USL} - \text{LSL}) / (6s); C_{pk} = \min(\text{USL} - \bar{x}, \bar{x} - \text{LSL}) / (3s)$$

Assumptions and units: Stable process; s estimates standard deviation; limits share measurement units.

Verification: Normal-process assumptions needed for simple tail probability interpretation; measurement uncertainty matters. [NIST]

E22 · CAM scallop screening

$$h = R - \sqrt{R^2 - (s/2)^2}$$

Assumptions and units: Ball cutter radius R ; step-over $s \leq 2R$; ideal local flat surface.

Verification: Real geometry, tool deflection, chatter, collision and post-processor require verification. [DERIVED]

E23 · Additive volumetric energy proxy

$$E_v = P / (v h t)$$

Assumptions and units: P W; scan speed v mm/s; hatch h mm; layer t mm; E_v J/mm³.

Verification: A proxy, not process qualification: equal E_v can produce different melt dynamics. [AM]

E24 · Product reuse metric

$$\text{reuse_rate} = N_{\text{qualified_reused}} / N_{\text{total}}$$

Assumptions and units: Define item granularity, requirements equivalence, version and qualified context.

Verification: 98% is a hypothesis until measured; identical shape does not mean identical qualification. [DERIVED]

E25 · Quantum variational objective

$$E(\theta) = \langle \psi(\theta) | H | \psi(\theta) \rangle; H = \sum_j c_j P_j$$

Assumptions and units: Normalized trial state; Pauli operators P_j ; c_j in energy units; finite-basis encoded Hamiltonian.

Verification: Ideal variational bound does not remove noisy measurement bias; no universal QPU acceleration claim. [QM]

E26 · Cable wave speed and impedance

$$ct = \sqrt{H/\mu L}; Z = \sqrt{H \mu L}$$

Assumptions and units: Ideal taut string, tension H N, mass per length μL kg/m; ct m/s; mechanical impedance Z N s/m.

Verification: Neglects bending stiffness and nonlinear geometry; use measured broadband dynamics for real corridors. [CONT]

E27 · Ideal terminal wave reflection

$$\Gamma = (Z_{\text{load}} - Z_{\text{line}}) / (Z_{\text{load}} + Z_{\text{line}})$$

Assumptions and units: Consistent mechanical wave convention and scalar impedances; energy reflection is $\text{abs}(\Gamma)^2$ for real positive impedances.

Verification: Matching one ideal line does not prove whole-network damping or resilience. [CONT]

E28 · Bulk charge relaxation and sheet resistance

$$\tau = \epsilon / \sigma_e = \epsilon \rho_e; R_{\text{sheet}} = \rho_e / t$$

Assumptions and units: Homogeneous Ohmic bulk medium; ρ_e ohm m; t m; R_{sheet} ohms per square.

Verification: Do not combine metal bulk resistivity with polymer permittivity to certify a multilayer lightning response. [EM]

E29 · Joule heating

$$E_J = \int I(t)^2 R(t) dt$$

Assumptions and units: Current A , resistance ohm, time s , energy J ; applies to resistive dissipation.

Verification: High-frequency current crowding, contact resistance, phase change and breakdown need coupled modelling. [EM]

E30 · Carnot upper bound

$$\eta_{\text{th}} \leq 1 - T_{\text{cold}}/T_{\text{hot}}; P_e = \eta_{\text{th}} \dot{Q}_{\text{hot}}$$

Assumptions and units: Reservoir temperatures K ; bound for heat-engine conversion between two thermal reservoirs.

Verification: Real geothermal cycle, pump loads, reservoir decline and availability reduce net performance. [THERMO]

E31 · Lorentz force and resistive MHD

$$\mathbf{f} = \mathbf{J} \times \mathbf{B}; \mathbf{J} = \sigma_e (\mathbf{E} + \mathbf{v} \times \mathbf{B}); q_J = \mathbf{J} \cdot \mathbf{J} / \sigma_e$$

Assumptions and units: Simple isotropic Ohmic MHD, $\mathbf{f}$ N/m³, $\mathbf{J}$ A/m², $\mathbf{B}$ T, q_J W/m³.

Verification: Electrode chemistry, Hall effects if relevant, magnetic power and hydraulic losses require additional models. [EM]

E32 · Inductive energy and discharge

$$E=0.5 L I^2; P=-dE/dt; I_{new}=\sqrt{I_{old}^2-2 \Delta E/L}$$

Assumptions and units: Constant positive inductance L H; ideal released energy ΔE between 0 and E_{old} .

Verification: If requested energy exceeds storage, reject the request; silently clamping current to zero violates delivered-energy accounting. [EM]

E33 · Circular toroid inductance

$$L=\mu_0 N^2 (R-\sqrt{R^2-a^2})$$

Assumptions and units: Ideal air-core continuously wound toroid, circular cross-section radius $a < R$, mean radius R , N turns.

Verification: Leakage, finite winding, field variation, support and superconducting constraints require full analysis. [EM]

E34 · Magnetic field energy density

$$u_B=B^2/(2 \mu_0); E=\int u_B dV$$

Assumptions and units: Vacuum or linear nonmagnetic approximation; B T; u_B J/m³.

Verification: Maximum local field cannot be used as a uniform volume average without justification. [EM]

E35 · Magnetic-support mass scaling

$$M_{\text{support}} = C_{\text{geom}} (\rho_{\text{material}}/\sigma_{\text{allow}}) E_{\text{magnetic}}$$

Assumptions and units: Dimensional scaling, not a universal certified lower bound; C_{geom} depends on load paths, stress state and geometry.

Verification: The source's coefficient-one inequality is not a complete virial structural design. [DERIVED]

E36 · Precipitation equilibrium and activities

$$IAP=\prod_i a_i^{\nu_i}; SI=\log_{10}(IAP/K_{sp})$$

Assumptions and units: Activities dimensionless relative to a standard state; consistent thermodynamic reaction and database temperature.

Verification: For saline brine use an appropriate activity model. Concentration-only K_{sp} and fixed pH do not determine yield. [GEOCHEM]

E37 · Trapezoidal channel geometry

$$A=y(b+zy); P_w=b+2y\sqrt{1+z^2}; R_h=A/P_w$$

Assumptions and units: Bottom width b m, depth y m, side slope z horizontal/vertical, wetted perimeter P_w m.

Verification: Hydraulic radius is not hydraulic depth; use actual geometry, sediment and lining condition. [CHANNEL]

E38 · Manning head loss, SI

$$v=(1/n) Rh^{2/3} \sqrt{Sf}; hf=n^2 L Q^2/(A^2 Rh^{4/3})$$

Assumptions and units: Manning n has units s/m^{1/3}; SI convention; quasi-uniform friction model.

Verification: Backwater, gates, free-surface transitions and changing depth require spatial hydraulic solution.
[CHANNEL]

E39 · Series channel and twin-tunnel flow

$$\Delta H=Q^2 [(K+f L t/Dt)/(2g A t^2)+n^2 L o/(A o^2 R h^{4/3})]$$

Assumptions and units: At is combined area of identical parallel tunnels; Ao canal area; K referenced to tunnel velocity Q/At; same total Q in series.

Verification: Different areas require different velocities. This constant-section screening equation is not a full backwater model. [DERIVED]

E40 · Lake water equilibrium

$$dV/dt=Q_{in}-Q_{out}+A(P-E)-Q_{seep}; A_{eq}=Q_{in}/(E-P)$$

Assumptions and units: Consistent time units; last expression assumes steady state, no outlet/seepage and spatially uniform E>P.

Verification: Lake hypsometry, salt accumulation and seasonal climate prevent a universal fixed area prediction.
[DERIVED]

E41 · Latent heat transport

$$P_{latent}=\rho_w Q_{evap} L_v$$

Assumptions and units: rho kg/m³, Q m³/s, L_v J/kg; P W.

Verification: Energy transfer into water vapour does not prescribe a regional temperature change or create net cooling of the planet. [THERMO]

E42 · Atmospheric column moisture balance

$$\partial_t W + \text{div}(F_q)=E-P; P=E-\partial_t W-\text{div}(F_q)$$

Assumptions and units: W kg/m²; F_q=(1/g) Integral q v dp, kg/(m s); E,P kg/(m² s); outward flux divergence convention.

Verification: The source image 164 has the wrong sign for outward divergence and omits storage; cloud-water terms may also matter. [ATM]

E43 · Soil carbon stock dynamics

$$dC/dt = I - k C; C(t) = I/k + (C_0 - I/k) \exp(-k t)$$

Assumptions and units: Constant I stock/time and $k > 0$ inverse time; C stock on a stated area/depth basis.

Verification: For wt%, convert stock using bulk density and sampled depth; calibrated dynamics do not follow from assumed asymptotes. [DERIVED]

E44 · Periodic soil thermal penetration

$$d = \sqrt{2 \alpha / \omega}; \alpha = k / (\rho c_p); T_{amp}(z) = T_{amp}(0) \exp(-z/d)$$

Assumptions and units: Homogeneous semi-infinite diffusion model with sinusoidal boundary, ω rad/s.

Verification: Surface temperature amplitude is imposed here; changing d alone does not prove a smaller surface amplitude. [CONT]

E45 · Spectral index and salinity regression

$$I = (\rho_a - \rho_b) / (\rho_a + \rho_b); EC = a_0 \exp(a_1 I) + a_2 \rho_c / \rho_d$$

Assumptions and units: Reflectances dimensionless; denominators nonzero; a_0, a_2 carry EC units; a_1 dimensionless.

Verification: Empirical source-specific index, not universal root-zone retrieval. Requires ground truth, calibration and uncertainty. [DERIVED]

E46 · Soil carbon to CO2 mass

$$mCO_2 = mC \cdot MCO_2 / MC \text{ approximately } (44.01/12.01) mC$$

Assumptions and units: Mass of carbon and CO₂ consistent; molar masses g/mol; chemical conversion only.

Verification: Sequestration claims need baseline, leakage, permanence, sampling depth and lifecycle emissions. [DERIVED]

E47 · Pile shaft resistance screening

$$R_s = \text{Integral shaft } \tau_s dA; \text{ for uniform shaft } R_s = \pi D L \tau_s$$

Assumptions and units: D, L m; shaft resistance τ_s Pa; R_s N for one pile.

Verification: Geotechnical factors, installation, rock joints, group effects and tension loading require project-specific design. [CONT]

E48 · Linear laminate mechanics

$$[N; M] = [A \ B; B \ D] [\epsilon_0; \kappa] - [N_{thermal}; M_{thermal}]$$

Assumptions and units: Classical lamination assumptions; membrane resultants N N/m and moments M N; stiffness from oriented ply data.

Verification: Strength, damage, transverse shear, creep, joints and pressure-pipe qualification require further models. [MAT]

E49 · Explicit heat-equation benchmark

$$\theta_i^{n+1} = \theta_i^n + Fo(\theta_{i-1}^n - 2\theta_i^n + \theta_{i+1}^n); Fo = \alpha \, dt/dx^2 \leq 1/2$$

Assumptions and units: Uniform 1D constant-diffusivity grid, forward Euler centred differences, fixed end temperatures.

Verification: Analytic sine-mode reference $\theta = \exp(-\pi^2 \alpha t/L^2) \sin(\pi x/L)$ enables a convergence test. [NUM]

E50 · Ideal net-lift envelope sizing

$$V_{\text{per_length}} = m_{\text{per_length}}(1 + \text{margin})/(\rho_{\text{air}} - \rho_{\text{gas}}); D_{\text{env}} = \sqrt{4 V_{\text{per_length}}/\pi}$$

Assumptions and units: Circular continuous-envelope approximation; $V_{\text{per_length}}$ m²; complete carried line mass kg/m; positive density difference.

Verification: Envelope/tether/ballast mass and wind cannot be omitted in a final sizing calculation. [DERIVED]

E51 · Rigid-body pose and tool transform

$$p_{\text{world}} = R p_{\text{tool}} + t; R^T R = I; \det R = 1$$

Assumptions and units: SE(3) pose with rotation R and translation t in consistent length units.

Verification: Calibrate robot/tool/workpiece frames; display accuracy does not establish pose accuracy. [ROBOT]

E52 · Robot Jacobian and velocity

$$v_{\text{task}} = J(q) \dot{q}$$

Assumptions and units: Joint coordinates and task-velocity convention fixed; angular and linear components distinguished.

Verification: Check singular values, joint limits, collisions and acceleration/torque capacity. [ROBOT]

E53 · Kinematic error propagation

$$\Sigma_{\text{pose}} \text{ approximately } J \Sigma_{\text{joint}} J^T + \Sigma_{\text{calibration}}$$

Assumptions and units: First-order covariance model with consistent coordinates; omitted cross-covariances assumed zero.

Verification: Near singularity and nonlinear errors require more complete propagation. [NIST]

E54 · State-space control

$$\dot{x} = A x + B u; y = C x + D u$$

Assumptions and units: Linearized model about a specified operating point, units and sampling stated.

Verification: A stable nominal model does not prove robustness to delays, saturation or unmodelled dynamics. [CONTROL]

E55 · Discrete Kalman measurement update

$$K = P_{\text{minus}} H^T (H P_{\text{minus}} H^T + R)^{-1}; x_{\text{plus}} = x_{\text{minus}} + K(y - H x_{\text{minus}})$$

Assumptions and units: Linear observation model and nonsingular innovation covariance; classical optimality assumes Gaussian noise.

Verification: Bias, correlation and changing noise models require diagnostics; filter confidence is conditional. [CONTROL]

E56 · Geometric chord tolerance

$$\delta = R[1 - \cos(\pi/N)]$$

Assumptions and units: Regular N-sided circular approximation, $N \geq 3$, radius R, maximum radial chord deviation δ .

Verification: Select tessellation by a specified geometric tolerance; manufacturing uncertainty is additional. [DERIVED]

E57 · CNC material removal rate

$$MRR = a_p a_e v_f$$

Assumptions and units: Axial depth a_p mm, radial engagement a_e mm, feed v_f mm/min; MRR mm³/min for a simple milling estimate.

Verification: Tool force, spindle power, chatter, wear and actual engagement limit productivity. [DERIVED]

E58 · Yield and scrap propagation

$$N_{\text{started}} = \text{ceil}(N_{\text{required}}/Y); C_{\text{accepted}} = C_{\text{total}}/N_{\text{accepted}}$$

Assumptions and units: $0 < Y \leq 1$ estimated yield; actual accepted count from inspection.

Verification: Correlated failures, rework and uncertain yield require scenario ranges rather than a guaranteed unit cost. [DERIVED]

Verification actually performed

Selected symbolic checks: {"PN_force_free": true, "PN_forced_polynomial": true}. See test-results.txt for numerical and geometry tests. No experimental validation or full equation-by-equation CAS certification.

References

[BIPM] BIPM. The International System of Units, SI Brochure, 9th edition. Defining constants and unit conventions. <https://www.bipm.org/en/publications/si-brochure>

[NIST] NIST. Technical Note 1297, Appendix A: Law of Propagation of Uncertainty. For process capability also use a validated statistical process-control method; TN1297 itself concerns measurement uncertainty. <https://www.nist.gov/pml/nist-technical-note-1297/nist-tn-1297-appendix-law-propagation-uncertainty>

[NUM] Standard mathematical references: Trefethen and Bau, Numerical Linear Algebra (SIAM, 1997); Nocedal and Wright, Numerical Optimization, 2nd ed. (Springer, 2006); Matiyasevich, Hilbert's Tenth Problem (MIT Press, 1993). These are bibliographic background references, not an exhaustive literature review.

[FEM] Brenner and Scott, The Mathematical Theory of Finite Element Methods, 3rd ed. (Springer, 2008), for variational foundations; MFEM official documentation for implementation capabilities. <https://mfem.org/>

[CONT] Landau and Lifshitz, Theory of Elasticity, 3rd ed. (1986), and Fluid Mechanics, 2nd ed. (1987). Standard continuum-mechanics reference equations; the selected application still requires its own boundary-value formulation.

[THERMO] Callen, Thermodynamics and an Introduction to Thermostatistics, 2nd ed. (1985), for balances and thermodynamic principles. The desalination expression is an idealized integration of concentration-dependent osmotic pressure, not a certified seawater equation of state.

[QM] David Tong, Quantum Mechanics and Topics in Quantum Mechanics, University of Cambridge lecture notes; variational principles and Born-Oppenheimer modelling. <https://www.damtp.cam.ac.uk/user/tong/qm/qm.pdf>

[QE] Quantum ESPRESSO official documentation and pw.x input description. Electronic-structure methods and convergence controls, not guaranteed predictive error. https://www.quantum-espresso.org/Doc/INPUT_PW.html

[MD] LAMMPS official documentation, compute stress/atom. Stress/pressure sign convention and kinetic/virial contributions. Verlet integration is a standard numerical method. https://docs.lammps.org/compute_stress_atom.html

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[MAT] Gurtin, Fried and Anand, The Mechanics and Thermodynamics of Continua (2010); Cahn and Hilliard, Free Energy of a Nonuniform System I, Journal of Chemical Physics 28, 258 (1958); Anderson, Fracture Mechanics, 4th ed. (2017). Constitutive parameters and empirical kinetics must be separately sourced for each material.

[FAO] Allen, Pereira, Raes and Smith. FAO Irrigation and Drainage Paper 56, Crop Evapotranspiration (1998), Chapters 2 and 5. <https://www.fao.org/4/x0490e/x0490e06.htm>

[FLUID] Swamee and Jain, Explicit equations for pipe-flow problems, Journal of the Hydraulics Division 102(5), 657-664 (1976); standard Darcy-Weisbach and transient-flow mechanics.

[AM] NIST, Additive Manufacturing Part Qualification. Energy-density proxies do not establish qualified process windows. <https://www.nist.gov/programs-projects/additive-manufacturing-part-qualification>

[EPO] European Patent Office. EP post-grant events and validity searching; Espacenet. Territorial legal status and additional searches matter. <https://www.epo.org/en/searching-for-patents/helpful-resources/patent-knowledge-news/ep-post-grant-events-and-validity>

[EU-M] EUR-Lex. Machinery safety requirements: Regulation (EU) 2023/1230, application from 20 January 2027. Scope and transitional details require product-specific review.

<https://eur-lex.europa.eu/EN/legal-content/summary/machinery-safety-requirements.html>

[EU-P] EUR-Lex. Directive 2014/68/EU on pressure equipment. Consult the consolidated/current applicable text and product scope. <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32014L0068>

[ISO] ISO. ISO 10791-1:2015 catalogue entry, geometric tests for machining centres. This public entry is not the full licensed standard. <https://www.iso.org/standard/54729.html>

[GEOCHEM] USGS PHREEQC documentation, speciation, activities, reaction and equilibrium calculations. Database and activity model must match the solution and phase system.
<https://water.usgs.gov/water-resources/software/PHREEQC/documentation/phreeqc3-html/phreeqc3-2.htm>

[CHANNEL] US Bureau of Reclamation, Water Measurement Manual, hydraulic radius and slope-area methods. SI Manning convention is stated in this supplement.
https://www.usbr.gov/tsc/techreferences/mands/wmm/chap02_11.html

[ATM] Column conservation law derived from moisture mass conservation with outward flux sign. ECMWF background on water and energy conservation in atmospheric models. <https://www.ecmwf.int/en/newsletter/172/news/fixing-water-and-energy-budget-imbalances-integrated-forecasting-system>

[GRAV] David Tong, General Relativity, Chapter 4, Einstein equations and Friedmann cosmology. These models are included as domain boundaries and are not implemented in the prototype.
<https://www.damtp.cam.ac.uk/user/tong/gr/grhtml/S4.html>

[ROBOT] Lynch and Park, Modern Robotics: Mechanics, Planning, and Control (2017), rigid transforms and Jacobians.
<https://modernrobotics.northwestern.edu/>

[CONTROL] Astrom and Murray, Feedback Systems: An Introduction for Scientists and Engineers (2008), linear systems, feedback and estimation. <https://fbsbook.org/>

[DERIVED] Algebraic derivation or unit conversion in this supplement. Assumptions and applicable domain are part of the equation record.

Appendix: 152 embedded source formula/symbol images

Faithful transcriptions, including source errors and unverified values. These are not additional validated equations. Use the review references for corrections and applicability.

image22.png | source paragraph(s) 8712

2.0 m

Review: Source symbol or scenario input; no independent validation implied

image23.png | source paragraph(s) 8712, 8943

$h = 200 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image24.png | source paragraph(s) 8712

500 m

Review: Source symbol or scenario input; no independent validation implied

image25.png | source paragraph(s) 8712

$v_w = 10 \text{ to } 45 \text{ m/s}$

Review: Source symbol or scenario input; no independent validation implied

image26.png | source paragraph(s) 8715

c_t

Review: Source symbol or scenario input; no independent validation implied

image27.png | source paragraph(s) 8715

H_0

Review: Source symbol or scenario input; no independent validation implied

image28.png | source paragraph(s) 8715

m_{total}

Review: Source symbol or scenario input; no independent validation implied

image29.png | source paragraph(s) 8716

$c_t = \sqrt{H_0/m_{\text{total}}} = \sqrt{1.006\text{e}8 \text{ N} / 3283 \text{ kg/m}}$ approximately 175.05 m/s

Review: E26

image30.png | source paragraph(s) 8716

$Z_{\text{line}} = \sqrt{H_0 m_{\text{total}}} = \sqrt{1.006e8 \times 3283}$ approximately 5.747e5 N s/m

Review: E26

image31.png | source paragraph(s) 8717

200 m

Review: Source symbol or scenario input; no independent validation implied

image32.png | source paragraph(s) 8717

$Z_{\text{damper}} = Z_{\text{line}}$

Review: Source symbol or scenario input; no independent validation implied

image33.png | source paragraph(s) 8717

$\Gamma_{\text{reflection}} = (Z_{\text{damper}} - Z_{\text{line}}) / (Z_{\text{damper}} + Z_{\text{line}})$ approximately 0

Review: E27

image34.png | source paragraph(s) 8718

greater than 92%

Review: Source symbol or scenario input; no independent validation implied

image35.png | source paragraph(s) 8718

450,000 km

Review: Source symbol or scenario input; no independent validation implied

image36.png | source paragraph(s) 8720

543.6 GWe

Review: Source symbol or scenario input; no independent validation implied

image37.png | source paragraph(s) 8720

245 C

Review: Source symbol or scenario input; no independent validation implied

image38.png | source paragraph(s) 8720

C5H10

Review: Source symbol or scenario input; no independent validation implied

image39.png | source paragraph(s) 8720

$T_c = 238.6 \text{ C}$

Review: Source symbol or scenario input; no independent validation implied

image40.png | source paragraph(s) 8720

$P_c = 4.51 \text{ MPa}$

Review: Source symbol or scenario input; no independent validation implied

image41.png | source paragraph(s) 8723

$R \geq 65/100$

Review: Source symbol or scenario input; no independent validation implied

image42.png | source paragraph(s) 8723, 8961

46,346.5 m³/s

Review: Source symbol or scenario input; no independent validation implied

image43.png | source paragraph(s) 8725

Global Capital Solvency Ratio = $\Phi_{\text{annual,Noble}} / \text{CAPEX}_{\text{annual,global}} = 545.30 \text{ billion USD/year} / 59.45 \text{ billion USD/year}$ approximately 9.17

Review: E19/E20; ratio is not solvency

image44.png | source paragraph(s) 8932, 8941

LH2

Review: Source symbol or scenario input; no independent validation implied

image45.png | source paragraph(s) 8932

$D = 0.35 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image46.png | source paragraph(s) 8932

$T = 20.28 \text{ K}$

Review: Source symbol or scenario input; no independent validation implied

image47.png | source paragraph(s) 8932

$D = 2.0 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image48.png | source paragraph(s) 8932

$T = 303.15 \text{ K}$

Review: Source symbol or scenario input; no independent validation implied

image49.png | source paragraph(s) 8932

$\Delta T = 282.87 \text{ K}$

Review: Source symbol or scenario input; no independent validation implied

image50.png | source paragraph(s) 8935

$\Delta L / L_0$

Review: Source symbol or scenario input; no independent validation implied

image51.png | source paragraph(s) 8935

20.28 K

Review: Source symbol or scenario input; no independent validation implied

image52.png | source paragraph(s) 8935

$\alpha(T)$

Review: Source symbol or scenario input; no independent validation implied

image53.png | source paragraph(s) 8936

$\Delta L/L_0 = \text{integral from } T_{\text{ambient}} \text{ to } T_{\text{cryo}} \text{ of } \alpha(T) \text{ d}T$

Review: S07

image54.png | source paragraph(s) 8936

$303.15 \text{ K} \rightarrow 20.28 \text{ K}$

Review: Source symbol or scenario input; no independent validation implied

image55.png | source paragraph(s) 8936

$(\Delta L/L_0)_{Ti}$ approximately -1.72×10^{-3} ($-172.0 \text{ mm per } 100 \text{ m}$)

Review: S07; material data required

image56.png | source paragraph(s) 8937

(Delta L/L_0)_UHMWPE approximately -3.124×10^{-3} (-312.4 mm per 100 m)

Review: S07; material data required

image57.png | source paragraph(s) 8938

σ_{thermal}

Review: Source symbol or scenario input; no independent validation implied

image58.png | source paragraph(s) 8938

$\sigma_{\text{thermal}} = E \text{ abs}(\Delta L/L_0) = 114 \times 10^9 \text{ Pa} \times 0.00172$ approximately 196.08 MPa

Review: S07; uniaxial constraint only

image59.png | source paragraph(s) 8940, 8959

100 m

Review: Source symbol or scenario input; no independent validation implied

image60.png | source paragraph(s) 8940

plus/minus 350 mm

Review: Source symbol or scenario input; no independent validation implied

image61.png | source paragraph(s) 8940

$N > 1 \times 10^7$

Review: Source symbol or scenario input; no independent validation implied

image62.png | source paragraph(s) 8943

$I_{\text{peak}} \geq 200 \text{ kA}$

Review: Source symbol or scenario input; no independent validation implied

image63.png | source paragraph(s) 8947

$\tau_{\text{relax}} = \epsilon_0 \epsilon_r \rho_{\text{surface}}$

Review: E28/E29; not a lightning proof

image64.png | source paragraph(s) 8947

ρ_{surface} approximately $2.44 \times 10^{-8} \text{ ohm m}$

Review: E28; bulk resistivity, not sheet resistance

image65.png | source paragraph(s) 8947

τ_{relax} approximately $8.854\text{e-}12 \times 8.5 \times 2.44\text{e-}8$ approximately $1.83\text{e-}18$ s

Review: E28/E29; multilayer model invalid as a proof

image66.png | source paragraph(s) 8952

70.59 billion USD/year

Review: Source symbol or scenario input; no independent validation implied

image67.png | source paragraph(s) 8953

545.30 billion USD/year

Review: Source symbol or scenario input; no independent validation implied

image68.png | source paragraph(s) 8954

$\Phi_{\text{annual,Noble}} / \text{sum CAPEX}_{\text{global}} = 545.30 \text{ billion} / 70.59 \text{ billion}$ approximately 7.72 (source label: Fully Solvent Across All Red Zones)

Review: E19/E20; ratio is not solvency

image69.png | source paragraph(s) 8959

243.2 mm

Review: Source symbol or scenario input; no independent validation implied

image70.png | source paragraph(s) 8960

τ approximately $1.83\text{e-}18$ s

Review: Source symbol or scenario input; no independent validation implied

image71.png | source paragraph(s) 8961

18,400 m³/s

Review: Source symbol or scenario input; no independent validation implied

image72.png | source paragraph(s) 8961

27,946.5 m³/s

Review: Source symbol or scenario input; no independent validation implied

image73.png | source paragraph(s) 8961

545.30 billion USD/year

Review: Source symbol or scenario input; no independent validation implied

image74.png | source paragraph(s) 9390

450,000 line-km

Review: Source symbol or scenario input; no independent validation implied

image75.png | source paragraph(s) 9390, 9416

plus/minus 800 kV

Review: Source symbol or scenario input; no independent validation implied

image76.png | source paragraph(s) 9390

$\Delta P / \Delta t > 25 \text{ GW/s}$

Review: Source symbol or scenario input; no independent validation implied

image77.png | source paragraph(s) 9390

greater than 50 ms

Review: Source symbol or scenario input; no independent validation implied

image78.png | source paragraph(s) 9393

E_M

Review: Source symbol or scenario input; no independent validation implied

image79.png | source paragraph(s) 9393

R_maj

Review: Source symbol or scenario input; no independent validation implied

image80.png | source paragraph(s) 9393

r_min

Review: Source symbol or scenario input; no independent validation implied

image81.png | source paragraph(s) 9393

N

Review: Source symbol or scenario input; no independent validation implied

image82.png | source paragraph(s) 9393

I_{op}

Review: Source symbol or scenario input; no independent validation implied

image83.png | source paragraph(s) 9394

$E_M = (1/2) L_{toroid} I_{op}^2 = (\mu_0 N^2/2) (R_{maj} - \sqrt{R_{maj}^2 - r_{min}^2}) I_{op}^2$

Review: E32/E33

image84.png | source paragraph(s) 9395

$R_{maj} = 4.50 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image85.png | source paragraph(s) 9396

$r_{min} = 1.20 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image86.png | source paragraph(s) 9397

$B_{max} = 14.5 \text{ T}$

Review: Source symbol or scenario input; no independent validation implied

image87.png | source paragraph(s) 9398

u_B

Review: Source symbol or scenario input; no independent validation implied

image88.png | source paragraph(s) 9398

$u_B = B_{max}^2 / (2 \mu_0) = 14.5^2 / [2 \times (4 \pi \times 1e-7)]$ approximately $8.366e7 \text{ J/m}^3$ (83.66 MJ/m³)

Review: E34

image89.png | source paragraph(s) 9399

E_{node} approximately 12.5 MWh = $4.50e10 \text{ J}$

Review: E32; assumed energy capacity

image90.png | source paragraph(s) 9400

M_{struct}

Review: Source symbol or scenario input; no independent validation implied

image91.png | source paragraph(s) 9400

$M_{\text{struct}} \geq (\rho_{\text{struct}}/\sigma_{\text{allow}}) E_M$

Review: E35; no universal coefficient-one bound

image92.png | source paragraph(s) 9401

$\rho_c = 1500 \text{ kg/m}^3$

Review: Source symbol or scenario input; no independent validation implied

image93.png | source paragraph(s) 9401

$\sigma_{\text{allow}} = 850 \text{ MPa}$

Review: Source symbol or scenario input; no independent validation implied

image94.png | source paragraph(s) 9401

$M_{\text{struct}} \geq (1500/8.50\text{e}8) \times 4.50\text{e}10$ approximately 79,412 kg (79.4 metric tonnes)

Review: E35; arithmetic only

image95.png | source paragraph(s) 9404

873 million tonnes/year

Review: Source symbol or scenario input; no independent validation implied

image96.png | source paragraph(s) 9404, 9417

180 million hectares

Review: Source symbol or scenario input; no independent validation implied

image97.png | source paragraph(s) 9404

P

Review: Source symbol or scenario input; no independent validation implied

image98.png | source paragraph(s) 9404

K

Review: Source symbol or scenario input; no independent validation implied

image99.png | source paragraph(s) 9404

PO4(3-)

Review: Source symbol or scenario input; no independent validation implied

image100.png | source paragraph(s) 9404

K+

Review: Source symbol or scenario input; no independent validation implied

image101.png | source paragraph(s) 9404

104,133 m³/s

Review: Source symbol or scenario input; no independent validation implied

image102.png | source paragraph(s) 9408

$\text{Mg}^{2+} + \text{NH}_4^+ + \text{PO}_4^{3-} + 6 \text{H}_2\text{O} \leftrightarrow \text{MgNH}_4\text{PO}_4 \cdot 6\text{H}_2\text{O} \text{ (solid)}$

Review: E16/E36

image103.png | source paragraph(s) 9408

K'_{sp}

Review: Source symbol or scenario input; no independent validation implied

image104.png | source paragraph(s) 9408

25 C

Review: Source symbol or scenario input; no independent validation implied

image105.png | source paragraph(s) 9408

$K_{\text{sp}} = [\text{Mg}^{2+}][\text{NH}_4^+][\text{PO}_4^{3-}]$ approximately 7.08×10^{-14} (pK_{sp} approximately 13.15)

Review: E36; activities and convention required

image106.png | source paragraph(s) 9409

[Mg²⁺] approximately 2.35 g/L approximately 96.7 mM

Review: E36; Mg molar mass conversion

image107.png | source paragraph(s) 9409

1.85 million metric tonnes/year

Review: Source symbol or scenario input; no independent validation implied

image108.png | source paragraph(s) 9416

12.5 MWh

Review: Source symbol or scenario input; no independent validation implied

image109.png | source paragraph(s) 9416

less than 0.1%

Review: Source symbol or scenario input; no independent validation implied

image110.png | source paragraph(s) 9417

1.85 million tonnes/year

Review: Source symbol or scenario input; no independent validation implied

image111.png | source paragraph(s) 9637

$z_{\text{bed}} = -15.2 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image112.png | source paragraph(s) 9637

$L_{\text{canal}} = 380 \text{ km}$

Review: Source symbol or scenario input; no independent validation implied

image113.png | source paragraph(s) 9637

9500 km²

Review: Source symbol or scenario input; no independent validation implied

image114.png | source paragraph(s) 9640

v

Review: Source symbol or scenario input; no independent validation implied

image115.png | source paragraph(s) 9640

Q

Review: Source symbol or scenario input; no independent validation implied

image116.png | source paragraph(s) 9640

$\Delta H = z_{\text{ocean}} - z_{\text{lake}} = 0.0 - (-11.5) = 11.5 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image117.png | source paragraph(s) 9640, 9664

380 km

Review: Source symbol or scenario input; no independent validation implied

image118.png | source paragraph(s) 9641

$$Q = A v = A \sqrt{2 g \Delta H / [\sum K_{\text{minor}} + f_D L_{\text{tunnel}}/D_h + 2 g L_{\text{open}} n^2/R_h^{(4/3)}]}$$

Review: E39; replace common velocity with section-specific velocities

image119.png | source paragraph(s) 9642

$$L_{\text{open}} = 338 \text{ km}$$

Review: Source symbol or scenario input; no independent validation implied

image120.png | source paragraph(s) 9642

$$b = 45.0 \text{ m}$$

Review: Source symbol or scenario input; no independent validation implied

image121.png | source paragraph(s) 9642

$$z = 2.0$$

Review: Source symbol or scenario input; no independent validation implied

image122.png | source paragraph(s) 9642

$$y = 6.0 \text{ m}$$

Review: Source symbol or scenario input; no independent validation implied

image123.png | source paragraph(s) 9642

$$n = 0.013$$

Review: Source symbol or scenario input; no independent validation implied

image124.png | source paragraph(s) 9643

$$R_h = 4.76 \text{ m}$$

Review: E37

image125.png | source paragraph(s) 9643

$$A_{\text{open}} = 342.0 \text{ m}^2$$

Review: E37

image126.png | source paragraph(s) 9644

$$L_{\text{tunnel}} = 42 \text{ km}$$

Review: Source symbol or scenario input; no independent validation implied

image127.png | source paragraph(s) 9644

$D_{\text{tunnel}} = 12.0 \text{ m}$

Review: Source symbol or scenario input; no independent validation implied

image128.png | source paragraph(s) 9644

$f_D = 0.011$

Review: Source symbol or scenario input; no independent validation implied

image129.png | source paragraph(s) 9645

$A_{\text{tunnel}} = 2 \times (\pi/4) \times 12.0^2 = 226.2 \text{ m}^2$

Review: E39

image130.png | source paragraph(s) 9646

v_{canal} approximately 1.38 m/s

Review: E39; source result not a solved network

image131.png | source paragraph(s) 9646

$Q_{\text{gravity}} = 342.0 \text{ m}^2 \times 1.38 \text{ m/s}$ approximately 471.96 m³/s (4.72e5 L/s)

Review: E39

image132.png | source paragraph(s) 9647

$V_{\text{annual}} = 471.96 \text{ m}^3/\text{s} \times 3.1536 \times 10^7 \text{ s}$ approximately 1.488e10 m³/year (14.88 billion m³/year)

Review: E40

image133.png | source paragraph(s) 9649

E_{evap} approximately 2200 mm/year = 2.2 m/year

Review: Source symbol or scenario input; no independent validation implied

image134.png | source paragraph(s) 9650

$A_{\text{eq}} = V_{\text{annual}}/E_{\text{evap}} = 1.488 \times 10^{10} \text{ m}^3/2.2 \text{ m}$ approximately 6.764e9 m² approximately 6764 km²

Review: E40

image135.png | source paragraph(s) 9651

$\lambda = 2.45 \times 10^6 \text{ J/kg}$

Review: Source symbol or scenario input; no independent validation implied

image136.png | source paragraph(s) 9651

$P_{\text{cooling}} = (1.488 \times 10^{13} \text{ kg} \times 2.45 \times 10^6 \text{ J/kg}) / 3.1536 \times 10^7 \text{ s}$ approximately $1.156 \times 10^{12} \text{ W} = 1156 \text{ GWth}$

Review: E41; no direct temperature prediction

image137.png | source paragraph(s) 9651, 9665

-3.65 K

Review: Source symbol or scenario input; no independent validation implied

image138.png | source paragraph(s) 9653, 9896

180 million hectares

Review: Source symbol or scenario input; no independent validation implied

image139.png | source paragraph(s) 9656

theta

Review: Source symbol or scenario input; no independent validation implied

image140.png | source paragraph(s) 9656

EC_e

Review: Source symbol or scenario input; no independent validation implied

image141.png | source paragraph(s) 9657

$\text{NDSI} = (\rho_{2100\text{nm}} - \rho_{1600\text{nm}}) / (\rho_{2100\text{nm}} + \rho_{1600\text{nm}})$

Review: E45

image142.png | source paragraph(s) 9657

$\text{EC}_e = a_0 \exp(a_1 \text{NDSI}) + a_2 (\rho_{850\text{nm}} / \rho_{670\text{nm}})$

Review: E45; calibration missing

image143.png | source paragraph(s) 9664

-15.2 m

Review: Source symbol or scenario input; no independent validation implied

image144.png | source paragraph(s) 9664

471.96 m³/s

Review: Source symbol or scenario input; no independent validation implied

image145.png | source paragraph(s) 9664

6764 km²

Review: Source symbol or scenario input; no independent validation implied

image146.png | source paragraph(s) 9665

1156 GWth

Review: Source symbol or scenario input; no independent validation implied

image147.png | source paragraph(s) 9666

2.5 m

Review: Source symbol or scenario input; no independent validation implied

image148.png | source paragraph(s) 9872

1.8e6 km²

Review: Source symbol or scenario input; no independent validation implied

image149.png | source paragraph(s) 9872

less than 250 micrometres

Review: Source symbol or scenario input; no independent validation implied

image150.png | source paragraph(s) 9875

dC_{soil}/dt

Review: Source symbol or scenario input; no independent validation implied

image151.png | source paragraph(s) 9876

$$dC_{soil}/dt = I_{biomass}(t) - k_{microbial} C_{soil}(t) \exp(-E_a/(R T_{soil})) (\theta(t)/\theta_{field})^{\gamma}$$

Review: E43; carbon-stock basis required

image152.png | source paragraph(s) 9876

$I_{biomass} = 8.4 \text{ metric tonnes C/ha year}$

Review: Source symbol or scenario input; no independent validation implied

image153.png | source paragraph(s) 9877

θ/θ_{field} approximately 0.72

Review: Source symbol or scenario input; no independent validation implied

image154.png | source paragraph(s) 9878

2026 -> 2036

Review: Source symbol or scenario input; no independent validation implied

image155.png | source paragraph(s) 9879

$C_{\text{soil}}(0) = 0.18 \text{ wt\%} \rightarrow C_{\text{soil}}(10) = 3.65 \text{ wt\%}$

Review: E43; conflicts with source code at ten years

image156.png | source paragraph(s) 9880

$\lambda_{\text{dry}} = 0.25 \text{ W/(m K)} \rightarrow \lambda_{\text{wet, humus}} = 1.48 \text{ W/(m K)}$

Review: Source symbol or scenario input; no independent validation implied

image157.png | source paragraph(s) 9881

$d_{\text{damp}} = \sqrt{2 \lambda / (\rho c \omega)}$

Review: E44

image158.png | source paragraph(s) 9881

plus 142%

Review: Source symbol or scenario input; no independent validation implied

image159.png | source paragraph(s) 9881

$\Delta T_{\text{surface}} = 55 \text{ K}$

Review: Source symbol or scenario input; no independent validation implied

image160.png | source paragraph(s) 9881

$\Delta T_{\text{surface}} = 14 \text{ K}$

Review: Source symbol or scenario input; no independent validation implied

image161.png | source paragraph(s) 9883

14.48 billion m³/year

Review: E40; source says 14.48, earlier calculation says 14.88

image162.png | source paragraph(s) 9883, 9896

18,400 m³/s

Review: Source symbol or scenario input; no independent validation implied

image163.png | source paragraph(s) 9886

P_downwind

Review: Source symbol or scenario input; no independent validation implied

image164.png | source paragraph(s) 9887

$P_{\text{downwind}} = E_{\text{local}} + \text{div}(Q_{\text{vapor}}) = E_{\text{local}} + (1/g) \int_0^p (v \cdot \text{grad } q + q \cdot \text{div } v) dp$

Review: E42; sign and storage correction

image165.png | source paragraph(s) 9887

beta_recycle

Review: Source symbol or scenario input; no independent validation implied

image166.png | source paragraph(s) 9887, 9896

46.8%

Review: E42; unvalidated source percentage

image167.png | source paragraph(s) 9888, 9896

plus 215 mm/year

Review: E42; unvalidated source rainfall claim

image168.png | source paragraph(s) 9888

14.2%

Review: E42; unvalidated source reduction

image169.png | source paragraph(s) 9895

0.18%

Review: Source symbol or scenario input; no independent validation implied

image170.png | source paragraph(s) 9895

3.65%

Review: E43; ten-year code mismatch

image171.png | source paragraph(s) 9895

55 K

Review: Source symbol or scenario input; no independent validation implied

image172.png | source paragraph(s) 9895

14 K

Review: Source symbol or scenario input; no independent validation implied

image173.png | source paragraph(s) 9896

13,248 m3/s

Review: E42; prescribed scenario output